

Modular Patterns

1. Start with the sequence $2, 4, 8, 16, \dots$. Make a new sequence by taking the 1st digit base 5 of the first sequence. What is the 371st term in the new sequence?
2. How many of the digits $1, \dots, 6$ are the 1st digit base 7 of a square number?
3. How many of the digits $1, \dots, 6$ are the 1st digit base 7 of a cube?
4. What is the 1st digit base 11 of $9^{100}, 9^{101}, 9^{99}$?
5. What is the first digit base 8 of 5^{937} ? What are the first 3 digits base 2 of 5^{937} ?