Review guide for the final exam

This is a list of the sections from the text that will be covered on the final exam. For a more detailed list of the main topics for the sections through 7.4, take a look at the review sheets for exams 1-5.

Chapter 1. §1,2,3
Chapter 2. §1, 2, 3, 4
Chapter 3 §1, 2, 3
Chapter 5 §1, 2 3, 4 (§5 was not covered)
Chapter 6 §1,2
Chaper 7 §1, 2, 3, 4 (§5 and §6 were not covered)
Chapter 8 §1, 2 (we won't have §3 on the exam)

§8.1 Symmetric matrices. You should know the basic facts on the eigenvalues, eigenvectors and diagonalization of symmetric matrices:
1. all eigenvalues of a symmetric matrix are real
2. if A is symmetric, and v, w are eigenvectors with different eigenvalues, then $v \cdot w = 0$
3. For an $n \times n$ symmetric matrix A, $\mathbb{R}^n$ has an orthnormal basis of eigenvectors for A.
4. For an $n \times n$ symmetric matrix A, there is an $n \times n$ orthogonal matrix U that diagonalizes A: $U^{-1}AU = D$, D the diagonal matrix of eigenvalues of A.

§8.2. Quadratic forms. You should know: that for every quadratic form in n variables, $q(x_1, \ldots, x_n)$ there is a symmetric matrix A with

$$q(x) = x^T Ax; \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}.$$

You should also know how finding an eigenbasis for A allows you to diagonalize q: if $v_1, \ldots, v_n$ is an orthonormal eigenbasis for A and $Av_i = \lambda_i v_i$, then

$$q(c_1 v_1 + \ldots + c_n v_n) = \lambda_1 c_1^2 + \ldots + \lambda_n c_n^2.$$

Thus, you can tell if $q(x)$ is (a) always > 0, (b) always < 0 or (c) sometimes > 0 and sometimes < 0 (for $x \neq 0$) if (a) all $\lambda_i > 0$, (b) all $\lambda_i < 0$, (c) some $\lambda_i < 0$ and some $\lambda_i > 0$.