Math U550 Real Analysis, Fall 2007.

Syllabus.

Text: *Real Analysis — A Constructive Approach*, by Mark Bridger (John Wiley, 2006).

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Real analysis is the theoretical underpinnings of the calculus. We begin by studying how the integers and rationals are constructed from the whole numbers (Chapter 0 of the book). We then move on to examine the more difficult construction of the real numbers from the rationals (Chapter 1). Traditionally this has been done using Dedekind cuts, Cauchy sequences, or nested intervals. We will adopt a newer approach using interval arithmetic and families of intersecting and arbitrarily small rational intervals. We prove that the reals, as constructed, are complete. Next (Chapter 2), we prove an Inverse Function Theorem, using certain inequalities called Lipschitz conditions as hypotheses. We apply this to deduce the existence of nth roots, then the exponential function, and finally logarithms. Instead of studying pointwise continuity and differentiability, we introduce the corresponding uniform notions (Chapters 4 and 6), and show that all of the usual functions of calculus are uniformly continuous and differentiable on bounded intervals. We prove a mean value type inequality, the Law of Bounded Change. In between these two topics, we use the completeness of the reals to prove that uniformly continuous functions have Riemann integrals (Chapter 4). Finally, we deduce the Fundamental Theorem of Calculus and, as time permits, give further applications to other theorems of calculus (Chapter 5).

Grading: Weekly quizes (every Thursday), average constitutes 30%, midterm grade constitutes 30% of the grade, Final Examination - 40% of the grade.

Goals: In addition to covering the topics listed above, the other goals of MTHU550 are: (a) becoming acquainted with the nature of modern mathematical abstraction and (b) learning how to write careful, logical, and understandable mathematical proofs.