Riemann-Roch theorem for $\mathcal{D}$-modules

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Abstract. An elliptic pair on a complex manifold X is the data of a coherent $\mathcal{D}_X$-module $\mathcal{M}$ and an $\mathbb{R}$-constructible sheaf F such that the intersection of the characterictic variety of $\mathcal{M}$ and the microsupport of F is contained in the zero-section of T^*X. If this intersection is compact, then the cohomology of the complex of solutions $\mathrm{RHom}_{\mathcal{D}}(\mathcal{M} \otimes F, \mathcal{O}_X)$ is finite dimensional over $\mathbb{C}$ and its index $\chi(X;\mathcal{M},F)$ is given by the formula (Schapira-Schneiders):

$$\chi(X;\mathcal{M},F) = \int_{T^*X} \mu eu(\mathcal{M}) \cup \mu eu(F).$$

Here $\mu eu(\mathcal{M})$ is the microlocal Euler class of $\mathcal{M}$ and $\mu eu(F)$ is Kashiwara's microlocal Euler class of F.

In this talk, we shall explain the meaning of this formula and its links with classical Riemann-Roch and Atiyah-Singer theorems.