

ADVANCES IN THE THEORY OF SEARCH

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In the late 70-ties I. Wegener and I wrote the first book on search. Its title “Search problems” indicates the exploratory state of the subject at that time. Contributions (often rediscovered several times) came from Statistics, Economics, Computer Science or even entertainment games. We started viewing the subject as the part of Information Theory dealing with gain of information (the other two main tasks being storage and transfer of information). A longstanding very hard combinatorial problem of Dorfman runs under the name group testing. It originated in Medicine. Recently search engines in the internet gave further impetus for theoretical advances. One direction among them is called “guessing.” Actually, all scientific endeavors can be viewed as search! To convey the flavor of the subject to a broader mathematical community we present the solutions of two seemingly basic classical problems.

The Rényi - Ulam search game with t lies has an equivalent formulation in terms of transmission of messages over a noisy binary channel with t errors in the presence of feedback. For block length n and error fraction $\tau = t/n$, the optimal rate for large n is called the capacity-error function $C_2(\tau)$.

In his 1964 MIT thesis, Berlekamp found a coding strategy achieving equality of C_2 with the Hamming bound H_2 for infinitely many τ . He also showed that C_2 coincides with the tangent at H_2 through the point $(1/3, 0)$.

In joint work with C. Deppe and V. Lebedev we discovered a coding scheme which gives such results for all alphabet sizes q . Surprisingly, the corresponding tangent at H_q , the q -ary Hamming bound, starts for every $q \geq 3$ at $(1/2, 0)$!

In 1965 Rényi asked how a restriction, say $k = \kappa N$, on the tests size in non-adaptive search for one of N objects reduces the number $\log N$ of tests needed, if there are no lies. The ratewise optimality of the binary entropy bound $\log N/h(\kappa) + o(\log N)$ was missed in the literature and was also not asked for by him.