The growth of cohomology in towers of manifolds

In many topological or geometric contexts, one is given a tower of covering spaces

$$\cdots \to X_n \to X_{n-1} \to \cdots \to X_1 \to X_0$$

where the X_i are compact manifolds. Given such a tower, it is natural to ask about the behaviour of the Betti numbers (both the usual Betti numbers, and also the $\mathbb{F}_p$-Betti numbers for some prime p) of the X_i, and in particular, about their rate of growth as $i \to \infty$.

A well-known example of such a tower occurs when X_0 is the complement in S^3 of a tubular neighbourhood of a knot, and the tower is obtained by unwrapping a meridian encircling this tubular neighbourhood. In this case, studying the growth of the Betti numbers up the tower is closely related to studying the Alexander polynomial of the knot.

In our talk we will focus on the following situation: Let p be a prime, let $\mathbb{Z}_p$ be the ring of p-adic integers, and let $\mathrm{GL}_n(\mathbb{Z}_p)$ be the group of invertible $n \times n$ matrices of with entries in $\mathbb{Z}_p$. If we are given a map $\pi_1(X_0) \to \mathrm{GL}_n(\mathbb{Z}_p)$, then we may define X_i to be the Galois cover of X_0 whose group of deck transformations is equal to the image of $\pi_1(X_0)$ in $\mathrm{GL}_n(\mathbb{Z}/p^i)$.

Examples of this type naturally arise when X_0 is a locally symmetric space, of the form $\Gamma\backslash G/K$, where Γ is an arithmetic subgroup of the semi-simple Lie group G (with maximal compact subgroup K). If time permits, we will explain the relation of these examples to the p-adic Langlands program.

This is joint work with Frank Calegari.