

Algebra Guidelines for Graduate Program

November 28, 2006

1 Description of Courses

1.1 Algebra 1, course G111

The course should cover Linear and Multilinear algebra, Basic properties of Categories and Functors, Basic properties of Groups and Group actions

- Vector spaces, bases, matrices, linear maps, kernel, image.
- Scalar products and orthogonality, bilinear maps, dual space, quadratic forms, Sylvester theorem.
- Determinants, inverse of a matrix, rank, subdeterminants.
- Symmetric, Hermitian and unitary operators.
- Eigenvectors, eigenvalues, characteristic polynomial.
- Diagonalization, triangulation, Jordan normal form.
- Categories, Functors
- Tensor products of vector spaces: universality property and construction.
- Exterior and symmetric powers of a vector space, exterior and symmetric multiplication, contractions.
- Groups, subgroups, normal subgroups, cosets, quotient groups, group homomorphisms.
- Isomorphism theorems
- Actions of groups (G-sets), orbits.

1.2 Algebra 2, course G112

- Groups, subgroups, normal subgroups, cosets of a subgroup, quotient groups, exact sequences of groups.
- Sylow theorems and solvable groups.

- Examples: cyclic groups, symmetric groups, direct and semidirect products.
- Abelian groups and their homomorphisms, exact sequences of abelian groups.
- Classification of finite abelian groups, infinite abelian groups, free abelian groups, torsion and torsion-free abelian groups, infinite products and sums.
- Rings, ideals, commutative rings, subrings and quotient rings.
- Units and factorization. polynomial rings.
- Fields and finite fields.
- Domains, Euclidean domains, principal ideal domains.
- Unique factorization domains and the Gauss lemma, Unique factorization in polynomial rings.
- The category of modules over a ring, submodules and quotient modules, exact sequences and relations with Hom.
- Cyclic modules, free modules and projective modules.
- Linear algebra of Euclidean rings and principal ideal domains.
- Abelian groups as modules, structure of finitely generated abelian groups and of finitely generated modules over a principal ideal domain.
- Relations of modules to problems of factorization.

2 Algebra Qualifying Exams

2.1 Qualifying Algebra 1

- Vector spaces, bases, matrices, linear maps, kernel, image.
- Scalar products and orthogonality, bilinear maps, dual space, quadratic forms, Sylvester theorem.
- Determinants, inverse of a matrix, rank, subdeterminants.
- Symmetric, Hermitian and unitary operators.
- Eigenvectors, eigenvalues, characteristic polynomial.
- Diagonalization, triangulation, Jordan normal form.
- Tensor products of vector spaces: universality property and construction.
- Exterior and symmetric powers of a vector space, exterior and symmetric multiplication, contractions.
- Groups, subgroups, normal subgroups, cosets, quotient groups, group homomorphisms.
- Isomorphism theorems, actions of groups (G-sets), counting orbits.

2.2 Qualifying Algebra 2

- Groups, subgroups, normal subgroups, cosets of a subgroup, quotient groups, exact sequences of groups.
- Sylow theorems and solvable groups.
- Examples: cyclic groups, symmetric groups, direct and semidirect products.
- Abelian groups and their homomorphisms, exact sequences of abelian groups.
- Classification of finite abelian groups, infinite abelian groups, free abelian groups, torsion and torsion-free abelian groups, infinite products and sums.
- Rings, ideals, commutative rings, subrings and quotient rings.
- Units and factorization. polynomial rings.
- Fields and finite fields.
- Domains, Euclidean domains, principal ideal domains.
- Unique factorization domains and the Gauss lemma, Unique factorization in polynomial rings.
- The category of modules over a ring, submodules and quotient modules, exact sequences and relations with Hom.
- Cyclic modules, free modules and projective modules.
- Linear algebra of Euclidean rings and principal ideal domains.
- Abelian groups as modules, structure of finitely generated abelian groups and of finitely generated modules over a principal ideal domain.
- Relations of modules to problems of factorization.