

MTH G301 - Functional Analysis

Instructor:	Maxim Braverman	457 LA, ext. 8769
Class hours:	MW 7:30 - 9:00 PM	Location: 544 NI
Textbook:	"Essential results in Functional Analysis" by Robert J. Zimmer	
Supplementary reading:	"Methods of Modern Physics. I. Functional Analysis" by M. Reed and B. Simon	
Prerequisites:	Linear Algebra, Theory of Lebesgue integration.	
Grading:	Home assignments will be given weekly and will be a basis for your grade. Do not accumulate a backlog: if you do it would be very difficult to catch up.	
Resources on the Web:	I recommend the homepage of the course given by professor Shubin in 1996. He used the same book and the site contains solutions of some of the problems from the book. The URL is: http://mystic.math.neu.edu/courses/funcan/	

What is **Functional Analysis**?

Functional Analysis developed in 20th century from an idea to treat functions as points in *infinite-dimensional space*. The idea allows a miraculously successful use of rich geometric intuition when dealing with functions. It proved to be extremely fruitful in applications to differential equations, harmonic analysis, ergodic theory, group representations, quantum mechanics, economic models. From a formal point of view, functional analysis is a generalization of linear algebra to infinite-dimensional spaces.

Course

Description:

The aim of the course is to provide an introduction to essential results of Functional Analysis and some of its applications. The main prerequisite is the theory of Lebesgue integration, which is necessary mainly to understand examples. The main abstract facts can be understood independently. Proof of some important basic theorems about Hilbert and Banach spaces (e.g. Hahn-Banach Theorem, Open Mapping Theorem) will be omitted to allow more time for applications of the abstract technique.

The textbook has a good set of exercises. Most of them are not difficult. You should try to solve as many of them as you can. The examples are at least as important as theorems. So you have to familiarize yourself with as many examples as possible.

Main topics to be covered:

1. Introduction to Hilbert and Banach spaces (this topic is assumed known in Zimmer's book; you can use Reed and Simon's book for references).
2. Basics on operators in Banach and Hilbert spaces and operator topologies.
3. Compact operators. Peter-Weyl theorem for compact groups.
4. Spectral theory. Gelfand's theory of commutative C*-algebras. Mean ergodic theorem.
5. Fourier transforms and Sobolev embedding theorems.
6. Distributions and Elliptic operators.
7. Mathematical scheme of quantum mechanics (this topic is not covered in Zimmer's book).