FUNCTIONAL ANALYSIS
MTH G301, key # 76 774
Professor M.Shubin
Fall 2007

Textbook: *Essential Results of Functional Analysis*, by Robert J. Zimmer
The University of Chicago Press, Chicago, 1990

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Class meetings: Monday and Wednesday 5:50 – 7:20 p.m., Room TBA

Functional Analysis developed in 20th century from an idea to treat functions as points in an infinite-dimensional space. This idea allows a miraculously successful use of rich geometric intuition when dealing with functions. It proved to be extremely fruitful in applications to differential equations, harmonic analysis, ergodic theory, group representations, quantum mechanics, economics models.

The aim of the course is to provide an introduction to essential results of Functional Analysis and some of its applications. The main prerequisite is the theory of Lebesgue integration, which is necessary mainly to understand examples, but at some moments is used in the theory itself. However the main abstract facts can be understood independently. Proofs of some important basic theorems about Hilbert and Banach spaces (e.g. Hahn–Banach Theorem and Open Mapping Theorem) will be omitted to allow more time for applications of the abstract technique. The main topics which will be covered are:

1. Basics on operators in Banach and Hilbert spaces and operator topologies.

2. Convexity and fixed point theorems. Haar measure. Krein–Milman theorem on extreme points.

3. Compact operators. Peter–Weyl theorem for compact groups.

4. Spectral theory. Gelfand's theory of commutative C^*-algebras. Mean ergodic theorem.

5. Fourier transforms and Sobolev embedding theorems.

6. Distributions and elliptic operators.

7. Mathematical scheme of quantum mechanics.

The textbook has a good set of exercises. Most of them are not difficult. You should try to solve as many of them as you can. Keep in mind a very important role of examples. The examples are as important as theorems (if not more). So you have to familiarize yourself with as many examples as you can.

Home assignments will be given weekly and will be a basis for your grade. Do not accumulate a backlog: if you do, it would be difficult to catch up.

For an additional reading and to cover the above mentioned gaps (which exist in the textbook too) I recommend the book "Methods of Modern Mathematical Physics. I. Functional Analysis" by Michael Reed and Barry Simon, Academic Press. There are also very many other good books on Functional Analysis.