Algebra 1, Fall 2005

Syllabus

(Preliminary version, July 22, 2005)

Instructor: Alex Martsinkovsky, 471 Lake, x. 5510, alexmart at neu dot edu

Office hours: By appointment.

Texts: *Linear Algebra* by S. Lang (★★☆☆☆), *Algebra* by S. Lang (★★☆☆☆).

Supplementary reading: *Intro to Commutative Algebra* by M. Atiyah and I.G. Macdonald, (★★★★★), *Noncommutative rings* by I. Herstein (★★★★☆).

The course consists of two parts: linear algebra and general algebra.

Linear Algebra: Gauss-Jordan elimination, the general linear group, the Jordan normal form, tensor product of vector spaces, the Kronecker product of matrices, categories and functors, tensor, symmetric, and exterior algebra of a vector space, bilinear and quadratic forms, the A - D - E classification.

General Algebra: Rings, ideals, prime and maximal ideals, Zorn's Lemma, polynomial rings, formal power series rings, Chinese Remainder Theorem and applications, principal ideal domains and finite modules over them, unique factorization, the spectrum of a commutative ring, matrix rings, algebras, division algebras and simple algebras, the path algebra of a quiver, the Weyl algebra, the group algebra of a finite group, the numerical semigroup algebra, the Stanley - Reisner algebra of a simplicial complex, modules, exact sequences, pullbacks and pushouts, chain conditions, the Fitting Lemma, the Snake Lemma, the exact sequence associated with composition, local rings, Nakayama's Lemma, the Hom-functor, the tensor product, adjoint functors, restriction, extension, and coextension of scalars, Dehn's solution to Hilbert's 3rd Problem, Gaussian integers, partial fraction decomposition, semisimple rings, Wedderburn's theorems, homological algebra, complexes, chain homotopy, projectives and injectives, mapping cone and mapping cylinder, homology, Ext and Tor, the Artin - Rees Lemma. Other topics if time permits.