QUALIFYING EXAM. ALGEBRA CONCENTRATION. January 2006.

1. Describe the conjugacy classes in the dihedral group D_6 of the symmetries of regular hexagon.

2. Let $\mathbf{K}$ be a field, X-an independent variable over $\mathbf{K}$. Let $A = \mathbf{K}[X]$ be the category of $\mathbf{K}[X]$-modules. We denote by A the ring A considered as a module over itself, and by B the field $\mathbf{K}(X)$ of rational functions in X considered as an A-module by left multiplication.

Which of the functors $Ab \to Ab$ are exact: $Hom_A(-, A)$, $Hom_A(-, B)$, $Hom_A(A, -)$, $Hom_A(B, -)$?

3. Describe the generators of the ideal in $\mathbf{K}[X, Y, Z]$ which is the kernel of the homomorphism

$$\phi : \mathbf{K}[X, Y, Z] \to \mathbf{K}[T]$$

given by $\phi(X) = T^2$, $\phi(Y) = T^3$, $\phi(Z) = T^4$. 4. Describe the structure of a 3-Sylow subgroup in the symmetric group S_6. How many of the 3-Sylow subgroups in S_6 do we have.

5. Describe the tensor products of the following abelian groups.

a) $\mathbf{Z}/m\mathbf{Z} \otimes \mathbf{Z}/n\mathbf{Z}$,

b) $\mathbf{Z} \otimes \mathbf{Z}/m\mathbf{Z}$,

c) $\mathbf{Z}(p^\infty) \otimes \mathbf{Z}(p^\infty)$.

6. Three matrices below give presentations of abelian groups (treated as $\mathbf{Z}$-modules) by generators and realtions. What are the groups ?

a) $\begin{pmatrix} 2 & 2 \\ 3 & 5 \end{pmatrix}$,

b) $\begin{pmatrix} 1 & 0 \\ 3 & 6 \\ 2 & 4 \end{pmatrix}$,

c) $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$,