# QUALIFYING EXAM, GENERAL ALGEBRA, September 2005

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1. Describe all conjugacy classes of 6x6 matrices $A$ with minimal polynomial $M_A(x) = x^2(x+8)$ and characteristic polynomial $P_A(x) = x^4(x+8)^2$.

2. Let $V$ and $W$ be finite dimensional vector spaces over the field $K$. Show that there is a natural isomorphism: $f : V^* \otimes W \to Hom_K(V, W)$.

3. Let $f : K^3 \to K^2$ be a linear operator with the matrix: $\begin{bmatrix} 4 & -1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$ in the coordinate bases. Find the kernel and the image of the linear operator:

$$\wedge^2 f : \wedge^2 K^3 \to \wedge^2 K^2$$

4. Find the rank and signature of the following quadratic form over $\mathbb{R}$:

$$Q(\underline{x}) = x_1x_2 - x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4.$$

5. Prove that any group of order 125 has nontrivial center.

6. Prove that any group of order $10 * 11^5$ is not simple.

7. Describe, up to isomorphism, all abelian groups of order 144, which do not have any elements of order 36.

8. Describe $Hom(\mathbb{Z}/5\mathbb{Z}, \mathbb{Z})$, the set of all group homomorphisms from the group $\mathbb{Z}/5\mathbb{Z}$ to the group $\mathbb{Z}$.

9. Please, justify your answer. Give an example of:

   a) An infinite nonabelian group.

   b) An infinite abelian group, with all of its elements of finite order (i.e. torsion group).

   c) A transitive action of a group on a set.

   d) A group homomorphism which is not one to one and is not onto.

   e) A group and its subgroup such that the left cosets are not the same as right cosets.

10. Please, justify your answer. Give an example of:

    a) A matrix which has two eigenvalues, but has 3 different Jordan blocks.

    b) A 3x3 matrix which is not diagonalizable over real numbers.

    c) A nilpotent matrix and its Jordan basis.