Northeastern University
Mathematics Department

Qualifying Exam, Algebra 2
Summer 2008

(1) Let A be a nonzero ring. Prove that the set of all prime ideals of A contains minimal (with respect to inclusion) elements.

(2) Suppose that any element x in a ring A satisfies $x^n = x$ for some $n > 1$ (depending on x). Prove that any prime ideal of A is maximal.

(3) Let $d \neq 0$ be a Gaussian integer. Express the number of elements in $\mathbb{Z}[i]/(d)$ in terms of d.

(4) How many units are there in the ring $\mathbb{Z}[i]/(70)$?

(5) Is $1/4$ divisible by 6 in $\mathbb{Z}(2^\infty)$? If yes – find the quotient, if not – explain why.