

Northeastern University
Mathematics Department

Qualifying Exam, Algebra 2
April - May 2009

- (1) Viewing \mathbb{C} as a two-dimensional vector space over \mathbb{R} , we endow it with the usual topology of \mathbb{R}^2 . Determine all continuous (in the above topology) field automorphisms of \mathbb{C} .
- (2) Determine the number of elements in the ring $\mathbb{Z}[\sqrt[3]{2}]/(2 + \sqrt[3]{2} + \sqrt[3]{4})$.
- (3) Let M be an artinian module (i.e., M has the DCC) over a commutative ring and f an endomorphism of M . Show that f is an isomorphism if and only if f is a monomorphism.
- (4) How many units are there in the ring $\mathbb{Z}[i]/(155)$?
- (5) Is $3/49$ divisible by 28 in $\mathbb{Z}(7^\infty)$? If yes – find the quotient, if not – explain why.