

Northeastern University
Mathematics Department

Qualifying Exam, Algebra 2
Spring 2009

- (1) Let G be the group of $n \times n$ non-singular matrices over $\mathbb{Z}/5$. What is the smallest value of n for which G contains an element of order 5?
- (2) Determine the number of elements in the ring $\mathbb{Z}[\sqrt{3}]/(4 + 2\sqrt{3})$.
Bonus question: Can you give a complete list of representatives of those classes? Explain why your list is complete.
- (3) How many units are there in the ring $\mathbb{Z}[i]/(287)$?
- (4) Is $5/27$ divisible by 39 in $\mathbb{Z}(3^\infty)$? If yes – find the quotient, if not – explain why.