

Northeastern University  
Mathematics Department

Qualifying Exam, ~~Algebra~~ <sup>Algebra</sup>  
Fall 2008

- (1) Let  $G$  be the group of  $n \times n$  non-singular matrices over  $\mathbb{Z}/7$ . What is the smallest value of  $n$  for which  $G$  contains an element of order 7?
- (2) How many elements are there in the ring  $\mathbb{Z}[\sqrt{5}]/(1 + \sqrt{5})$ ?  
**Bonus question:** Can you give a complete list of representatives of those classes? Explain why your list is complete.
- (3) General Gau will be hosting his friend General Tso. He has a habit of organizing elaborate ceremonies during such visits. In particular, he likes military parades with soldiers marching in square formations of the **same size** (of more than one soldier). But this time there is a difficulty: because of a bird flu epidemic, on the day of the parade some soldiers may be sick and not available for the parade. While General Gau cannot know the exact number of sick soldiers in advance, he is sure that it will be between 1 and 20. Prove that if he has sufficiently many soldiers, then he can indeed organize a parade with square formations. (The size of the squares will depend on the number of sick soldiers: for example, nine soldiers can be arranged in one square of size 3, whereas eight soldiers would require two squares of size 2).
- (4) How many units are there in the ring  $\mathbb{Z}[i]/(190)$ ?
- (5) Is  $1/9$  divisible by 15 in  $\mathbb{Z}(3^\infty)$ ? If yes – find the quotient, if not – explain why.