

NORTHEASTERN UNIVERSITY
Department of Mathematics
Qualifying Exam in Analysis, Part 1
September 2009

Please provide proofs of all your statements, except the ones which are proved in the textbook (in which case you should explicitly refer to such a statement).

1. For any fixed $a > 0$ investigate whether the expression

$$\lim_{n \rightarrow \infty} n(a^{\frac{1}{n}} - 1)$$

has the limit as $n \rightarrow \infty$, and find this limit whenever it exists.

2. Find $f'(x)$, $f''(x)$ for the function $f(x) = x^x$, where $x > 0$.

3. Calculate

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}.$$

4. Calculate

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n} \right).$$

5. Let us consider a subset $A \subset \mathbf{R}$, $A = (0, 1) \cup (1, 2)$.

- a) Is A complete?
- b) Is A compact?
- c) Is A connected?

6. Find the volume of the body in \mathbf{R}^3 defined by the inequality

$$\{(x, y, z) : x^2 + y^2 + z^2 + xy + xz + yz \leq 1\}.$$