

Northeastern University, Mathematics Department.

Analysis 2: Qualifying Exam

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Be sure to include your reasoning – no credit for unexplained answers.

1.) Let $\langle f_n \rangle$ be a sequence of continuous function defined on \mathbf{R} . Show that the set C of points where this sequence converges is an $F_{\sigma\delta}$.

2.) Let the integral $\int_a^b f(x)dx$ be convergent. Let $\langle v_n \rangle$, $n = 1, 2, \dots$ be a sequence of measurable and bounded functions on (a, b) (say $|v_n(x)| \leq c_n$ for all $x \in (a, b)$). Suppose that $\sum_{n=1}^{\infty} v_n(x)$ is uniformly convergent on (a, b) . Show that

$$\int_a^b f(x) \sum_{n=1}^{\infty} v_n(x) dx = \sum_{n=1}^{\infty} \int_a^b f(x) v_n(x) dx$$

3.) Let

$$\alpha = \frac{1}{2\pi} \frac{xdy - ydx}{x^2 + y^2}$$

Prove that α is a closed 1-form on $\mathbf{R}^2 - \{0\}$. Compute the integral of α over the unit circle S^1 . How does this result show that α is not exact? How does this show that $\delta i(\alpha)$ is not exact, where $i : S^1 \rightarrow \mathbf{R}^2$ is the canonical embedding?

4.) Solve the boundary value problem

$$u_t = u_{xx}, \quad 0 < x < \pi, \quad t > 0$$

$$u(0, t) = u(\pi, t) = 0$$

$$u(x, 0) = 4 \sin(4x) \cos(2x)$$

5.) Let $f \in L^p = L^p[0, 1]$. Suppose that $\Delta = \{\xi_0, \dots, \xi_n\}$ is a subdivision $0 = \xi_0 < \xi_1 < \dots < \xi_{n-1} < \xi_n = 1$ of $[0, 1]$, and that δ is the length of the largest subinterval in Δ . Let ϕ_Δ be the Δ -approximant to f . Show that ϕ_Δ converges to f in measure as $\delta \rightarrow 0$.