

NORTHEASTERN UNIVERSITY
DEPARTMENT OF MATHEMATICS

ANALYSIS 2 QUALIFYING EXAM

SEPTEMBER 2008

1. If $f \geq 0$ and $\int_E f d\mu = 0$, prove that $f(x) = 0$ almost everywhere on E .
2. Let $\{f_n\}$ be a sequence of measurable functions, and suppose that for every $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} \mu(\{x : |f_n(x)| > \varepsilon\}) < \infty.$$

show that $f_n \rightarrow 0$ almost everywhere.

3. Let $U \subset \mathbb{R}^n$ be an open bounded set with compact smooth boundary ∂U . Let ν denote the outward unit normal vector field on ∂U . Let f and g be smooth functions on \mathbb{R}^n . Use the Stokes theorem to show that

$$\int_{\partial U} f \frac{\partial g}{\partial \nu} d\mu_{n-1} = \int_U (f \Delta g + \nabla f \cdot \nabla g) d\mu_n,$$

where $\frac{\partial g}{\partial \nu} = \nabla \cdot \nu$ is the *normal derivative* of g and

$$\Delta g = \sum_{j=1}^n \frac{\partial^2 g}{\partial x_j^2} = \operatorname{div}(\operatorname{grad} f).$$

4. Give an example of a smooth one form on $\mathbb{R}^2 \setminus \{0\}$ which is closed but not exact.
5. Let $f(x) = |\sin x|$.
 - (a) Compute the Fourier series of f .
 - (b) Identify the pointwise limit of the Fourier series of f at each point $x \in \mathbb{R}$. Where is the convergence uniform?
 - (c) Show that, for $0 \leq x \leq \pi$,

$$\frac{\pi}{4} (\cos x - 1) + \frac{x}{2} = \sum_{j=1}^{\infty} \frac{\sin 2jx}{(2j-1)(2j)(2j+1)}.$$

Hint: Use the Fourier series of f' .