Mathematics Department, Northeastern University

Syllabus for Analysis 2 Qualifying Exam

Topics: Measure Theory; Integration and Function Spaces; Integration of Differential Forms; the Fourier transform.

References:
"Real Analysis", third edition, by H. L. Royden;
"Principles of Mathematical Analysis", third edition, by W. Rudin;
"Elements of the Theory of Functions and Functional Analysis", by A. N. Kolmogorov and S. V. Fomin;
"Introduction to Analysis", by Maxwell Rosenlicht;
"Fourier analysis and its applications", by G. B. Folland;
"Differential Forms with applications to the Physical Sciences", by H. Flanders.

σ-algebras, Borel sets, F_σ sets, G_δ sets; outer measure on $\mathbf{R}$, measurable set, Lebesgue measure; measurable function, simple function, Egoroff's theorem.

Riemann integral, Lebesgue integral, integrable function; Fatou's lemma, monotone convergence theorem, dominated convergence theorem, Riemann-Lebesgue lemma; functions of bounded variation, absolutely continuous functions; Lebesgue's theorem on the derivative of function of bounded variation; Jensen's inequality.

Product measure, Fubini's theorem on iterated integration; l^p and L^p; Minkowski, Young and Hölder inequalities; convergence in l^p and L^p, Riesz-Fischer theorem on completeness of L^p; bounded linear functionals, dual spaces, Riesz representation theorem.

Curves in $\mathbf{R}^2$, surfaces in $\mathbf{R}^3$; k-surfaces in $\mathbf{R}^n$; differential forms in $\mathbf{R}^n$; Leibnitz rule, change of variables; closed, exact forms; Stokes Theorem.

Fourier series: Bessel's inequality; criteria for pointwise convergence and convergence in L^2; completeness; applications to solving separable PDE's.

Fourier transform: invertibility and the inversion formula, Schwartz space, Plancherel theorem and Fourier transform in L^2, applications to ODE and PDE.