# GRADUATE COURSE ON HALL ALGEBRAS, QUIVERS AND QUANTUM GROUPS MTH G362–TOPICS IN ALGEBRA NORTHEASTERN UNIVERSITY, SPRING 2009

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The course will explore how a number of algebras which are familiar in representation theory (the algebra of symmetric functions, semisimple Lie algebras and their deformations known as quantum groups) can be realised as Hall algebras (a concept which the course will start by defining) of suitable abelian categories.

The abelian categories of interest are often module categories: modules over the polynomial ring in one variable $\mathbb{C}[x]$, over a quiver (i.e. an oriented graph) and, a little more geometrically, coherent sheaves over the projective line $\mathbb{P}^1$ or an elliptic curve defined over a finite field.

Such a link between a representation theoretic algebra $A$ and a module category is not only interesting in its own right, but sheds light on both sides. For example, a Hall algebra is always endowed with a natural basis and this gives rise to various interesting bases of symmetric functions, quantum groups and so on. With some additional work these can be upgraded to the canonical bases which figure very prominently in representation theory.

In the opposite direction, the structure constants of a Hall algebra encode interesting homological and geometric properties of the abelian category. For example the Euler characteristic of the quiver Grassmannians that Prof. Zelevinsky recently described in his GASC talk.

I intend the course to be entirely elementary and to require only a modest knowledge of homological algebra (abelian categories, Hom's and Ext's). A knowledge of (the structure theory of) semisimple Lie algebras and/or quantum groups will be be helpful here and there, but merely as an additional motivation to the results to be described, and will not be assumed. Similarly, although we will talk about coherent sheaves on $\mathbb{P}^1$ and an elliptic curve, I will not assume any knowledge of algebraic geometry and will review the necessary notions.

The main reference for the course will be the lectures notes [Sc]. Some additional sources covering related and further developments are listed below.

The course will be held on Tuesdays and Thursdays, from 6 to 7:30.

**The registration deadline is January 16**.

## References


[Cr] T. Cramer, *Double Hall algebras and derived equivalences*, arXiv:0809.3470.

[KS] K. Kremnizer, M. Szczesny, *Feynman graphs, rooted trees, and Ringel–Hall algebras*, arXiv:0806.1179.

[Re] M. Reineke, *The Harder–Narasimhan system in quantum groups and cohomology of quiver moduli*, Invent. Math. **152** (2003), 349–368, math.QA/0204059.

[Sc] O. Schiffmann, *Lectures on Hall algebras*, math.RT/0611617.

[SV] O. Schiffmann, E. Vasserot, *The elliptic Hall algebra, Cherednick Hecke algebras and Macdonald polynomials*, arXiv:0802.4001.