

READING COURSE ON STABILITY CONDITIONS
MTH G734–READINGS IN ALGEBRA
NORTHEASTERN UNIVERSITY, SPRING 2009

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The aim of this course will be to acquaint ourselves with T. Bridgeland’s space of stability conditions and to study it in a variety of examples of geometric and algebraic origin.

The notion of stability condition on an abelian or a triangulated category is a generalisation of the classical notion of stability in Geometric Invariant Theory (whose knowledge will not be assumed in the course). One novel feature, which is motivated by considerations in String Theory, is that rather than considering a preferred stability condition, one is led to consider the space of all such.

This space turns out to have the structure of a complex manifold and one can produce this way very interesting such manifolds. For example, starting with the category of sheaves on the minimal resolution of $\mathbb{C}^2/\mathbb{Z}^n$ one can construct the universal cover of the configuration space of n points in \mathbb{C} [Th2, Br2].

The aim of the reading course will be to acquaint ourselves with the general theory of stability conditions [Br1], and then carefully study a number of interesting examples: elliptic curves [Br1], quivers and Kleinian singularities [Br3, Th2], \mathbb{P}^2 [Br3] and possibly $K3$ surfaces [Br4].

The first two thirds part of the course will focus mostly on Bridgeland’s papers. Time, and audience’s desires permitting, we may then venture to explore other aspects of the theory such as for example the relation to Stokes phenomena [BTL].

The prerequisites for the course will be a knowledge of triangulated categories and a modicum of algebraic geometry.

The course will be run as a seminar given by graduate students. We will meet on Wednesdays, starting on February 4, from 10 to 1:30 in room 544 LA.

REFERENCES

- [Br1] T. Bridgeland, *Stability conditions on triangulated categories*, Ann. of Math. **166** (2007), 317–345, [math.AG/0212237](#).
- [Br2] T. Bridgeland, *Stability conditions and Kleinian singularities*, [math.AG/0508257](#).
- [Br3] T. Bridgeland, *Stability conditions on a non-compact Calabi-Yau threefold*, Comm. Math. Phys. **266** (2006), 715–733.
- [Br4] T. Bridgeland, *Stability conditions on $K3$ surfaces*, Duke Math. J. **141** (2008), 241–291, [math.AG/0307164](#).
- [BTL] T. Bridgeland, V. Toledano Laredo, *Stability conditions and Stokes factors*, [arXiv:0801.3974](#).
- [HRS] D. Happel, I. Reiten, S. Smalø, *Tilting in abelian categories and quasitilted algebras*. Mem. Amer. Math. Soc. **120** (1996), no. 575.
- [ST] P. Seidel, R. Thomas, *Braid group actions on derived categories of coherent sheaves*, Duke Math. J. **108** (2001), 37–108, [math.AG/0001043](#).
- [Th1] R. P. Thomas, *Derived categories for the working mathematician*, Winter School on Mirror Symmetry, Vector Bundles and Lagrangian Submanifolds (Cambridge, MA, 1999), 349–361, AMS/IP Stud. Adv. Math., 23, Amer. Math. Soc., Providence, RI, 2001, [math.AG/0001045](#).
- [Th2] R. P. Thomas, *Stability conditions and the braid group*, Comm. Anal. Geom. **14** (2006), 135–161.

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ORGANISATION OF THE COURSE

Week	Dates	Lectures	Speaker
4 Jan. 26–Feb. 1	Jan. 28	Organisational meeting	
5 Feb. 2–8	Feb. 4	Review of triangulated categories and tilting [Th1,HRS]	S. Pepin
6 Feb. 9–15	Feb. 11	Stability conditions on triangulated categories I [Br1]	A. Appel
7 Feb. 16–22			
8 Feb. 23–Mar. 1	Feb. 25	Stability conditions on triangulated categories II [Br1]	A. Appel
9 Mar. 2–8		<i>Spring break, no classes</i>	
10 Mar. 9–15			
11 Mar. 16–22			
12 Mar. 23–29	Mar. 25	Reflection functors [ST]	S. Stella
13 Mar. 30–Apr. 5	Apr. 1	Braid group actions on derived categories [ST,Th1]	S. Stella
14 Apr. 6–12			
15 Apr. 13–19	Apr. 15	Coherent sheaves on Kleinian singularities	S. Gautam
16 Apr. 20–26	Apr. 22	Stability conditions on Kleinian singularities [Th1,Br2]	S. Gautam
17 Apr. 27–May 3	Apr. 29	Stability conditions on local \mathbb{P}^2 [Br3]	S. Stella
to be decided		Stability conditions on K3 surfaces [Br4]	A. Appel
to be decided		Stability conditions and Stokes factors [BTL]	S. Gautam