MATH 7734–READINGS IN ALGEBRA
GEOMETRIC REPRESENTATION THEORY
NORTHEASTERN UNIVERSITY, FALL 2010

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Course description

The aim of this course is to understand how certain quantum groups such as quantised enveloping algebras and Hecke algebras can be realised geometrically as 'correspondences' acting on algebraic varieties.

These constructions generalise the following basic mechanism: if a group G acts on a variety V, then it acts on any vector space $H(V)$ naturally associated to V (for example $H(V)$ could be cohomology or K–theory) so the latter gives a representation of G and in good cases produces all its irreducible representations.

The course will be a blend of algebraic topology, algebraic geometry and representation theory. The prerequisites are a knowledge of semisimple Lie groups, basic (complex) algebraic geometry and algebraic topology, together with some notions of symplectic geometry (which can be aquired by reading chapter 1 of the reference book or attending Prof. Weitsman's course on symplectic geometry this term). The course is really meant to be an introduction to geometric representation theory which could lead to further reading and/or graduate courses on Nakajima quiver varieties for example.

The basic reference for the course is the book *Representation Theory and Complex Geometry* by N. Chriss and V. Ginzburg. The book is pretty ambitious, but the idea is to learn well a few basic and widely used tools and topics such as Borel–Moore homology, K–theory and the geometry of the flag and Steinberg varieties and use them to prove one such result: the affine Hecke algebra of a semisimple algebraic group G is isomorphic to the equivariant K–theory of its Steinberg variety. This is roughly the contents of chapters 3, 5 and 7 of the book.

The organisational meeting will take place on Wednesday, Sept. 8 in 544 Nightingale at 4:30 PM.

The course will be run as a seminar given by graduate students. We will meet twice a week, on Mondays and Wednesdays from 5:40 to 7:40 in 575 Lake, starting on Monday, Sept. 13.

Organisation of the course

Week	Dates	Topic	Speaker
1	Mon. Sept. 6: Labour day		
Sept. 6–12	Sept. 8	Organisational meeting	
2	Sept. 13	Borel–Moore homology, I	M. Balagovic
Sept. 13–19	Sept. 15	Borel–Moore homology, II	M. Balagovic
3	Sept. 20	Convolution in Borel–Moore homology, I	D. Jordan
Sept. 20–26	Sept. 22	Convolution in Borel–Moore homology, II	D. Jordan
4	Sept. 27	The flag variety, I	N. Bade
Sept. 27–Oct. 3	Sept. 29	The flag variety, II	N. Bade
5	Oct. 4	The nilpotent cone	S. Stella
Oct. 4–10	Oct. 6	The Steinberg variety	S. Stella
6	Oct. 11: Columbus day, no meeting		
	Oct. 12 or 13	Lagrangian construction of the Weyl group	S. Gautam
Oct. 11–17	Oct. 13 or 14	Geometric analysis of the $H(Z)$–action	S. Gautam
7	Oct. 18	Irreducible representations of Weyl groups	S. Gautam
Oct. 18–24	Oct. 20	Equivariant resolutions	A. Appel
8	Oct. 25	Basic K–theoretic constructions	A. Appel
	Oct. 27	The deformation construction	V. Toledano
Oct. 25–31		Specialization in equivariant K–theory	V. Toledano
9	Nov. 1	The Koszul complex and the Thom isomorphism	Y. Yang
	Nov. 3	Cellular fibration lemma	Y. Yang
Nov. 1–Nov. 7		The Künneth formula	Y. Yang
10	Nov. 8–10: Valerio in Eugene		
	Nov. 11 (Veterans' day)	Projective bundle theorem and Beilinson resolution	G. Zhao
		The Chern character	G. Zhao
	Nov. 12	The dimension filtration and devissage	G. Zhao
Nov. 8–14		The localization theorem	G. Zhao
11	Nov. 15	Functoriality	G. Zhao
	Nov. 17	Affine Weyl groups and Hecke algebras	S. Gautam
Nov. 15–21		Main theorems	
12	Nov. 22	Case $q = 1$: deformation argument, I	A. Appel
	Nov. 23	Case $q = 1$: deformation argument, II	A. Appel
Nov. 22–28	Nov. 24–28: Thanksgiving break		
13	Dec. 1	The Hecke algebra for SL_2	S. Stella
	Dec. 2	Proof of the main theorem, I	Y. Yang
Nov. 29–Dec. 5	Dec. 3	Proof of the main theorem, II	G. Zhao
14			
Dec. 6–12			