Do not write in the boxes immediately below.

| problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| points | | | | | | | | | | | | | | |

# MTH1124/1141/1724 Final Exam

March 12, 2002

Instructor's name____________________ Your name____________________

**Show all work. Answers straight from your calculator are worth zero points.**

**The number of total points on the exam is 100.**

1) (6 points each) Find the following indefinite integrals (i.e., anti-derivatives).

a) $\displaystyle\int \left(e^x + \frac{1}{\sqrt{x}} + 8\right) dx$

b) $\displaystyle\int \frac{3t^3 - 10t^2}{t}\, dt$

c) $\displaystyle\int \sec^2(7\theta)\, d\theta$

2) (6 points) A wavy spaghetti noodle, 10 centimeters long, is lying along the $x$-axis, with its left end at the origin. The cross-sections perpendicular to the $x$-axis are circles, whose radii are given by $e^{\sin x}$ at a distance of $x$ centimeters from the left end of the noodle. Estimate the volume of the entire noodle by using a Riemann sum with 5 subintervals of equal length and **right** endpoints of the subintervals. Round your answer to 4 decimal places, and be sure to include units.

3) (6 points) Solve the initial value problem $\dfrac{dy}{dx} = 2x\cos(x^2)e^{\sin(x^2)}$ with $y\left(\sqrt{\pi/2}\right) = e - 7$. (Hint: While the function on the right side looks bad, the problem is not very hard.)

4) (6 points) Find the area under the graph of $y = \dfrac{x^2}{\sqrt{3x^3+1}}$ and above the $x$-axis, where $1 \leqslant x \leqslant 2$.

5) (6 points) Evaluate the integral $\displaystyle\int_0^{\pi/2} \cos^3 x \sin x \, dx$.

6) (6 points) Find the average value of the function $f(x) = 3x^3 + 2x$ on the interval $1 \leqslant x \leqslant 3$.

7) (6 points) Use the following values of the function $f(x) = x + \cos x$, together with Simpson's Rule with 4 subintervals, to estimate the value of $\int_0^{3.2} (x + \cos x)\, dx$. Round your final answer to 4 decimal places.

**YOU WILL RECEIVE NO CREDIT ON THIS PROBLEM IF YOU USE A METHOD OTHER THAN SIMPSON'S RULE.**

| $x$ | 0 | 0.8 | 1.6 | 2.4 | 3.2 |
|---|---|---|---|---|---|
| $f(x)$ | 1.0000 | 1.4967 | 1.5708 | 1.6626 | 2.2017 |

8) (7 points) A circular cylinder of radius 4 meters and height 10 meters is filled with water to within 2 meters of the top (i.e., the bottom 8 meters of the tank are filled with water). Assume that water weighs 9800 Newtons per cubic meter. Write an integral for the amount of work required to pump all of the water to the top of the tank, and include units for the answer. **DO NOT EVALUATE THIS INTEGRAL.**

**Include units.**

9) (4 points each) A thin rod is lying along the $x$-axis, between $x = 1$ and $x = 4$, where $x$ is measured in meters. The density (actually, length-density) of the rod at coordinate $x$ is given by $\delta(x) = 1/x \quad kg/m$.

a) Find the mass of the rod. **Include units**.

b) Find the $x$-coordinate of the center of mass of the rod. **Include units**.

10) (6 points each) Consider the bounded region $R$ in the first quadrant, below the graph of $y = \dfrac{4}{x^3}$, above the line $y = 1/2$, and to the right of the line $x = 1$, where all distances are measured in meters.

a) Write an integral for the volume of the solid generated by revolving $R$ around the $x$-axis, and include units for the answer. **DO NOT EVALUATE THIS INTEGRAL.**

**Include units.**

b) Write an integral for the volume of the solid generated by revolving the same region $R$, as above, around the $y$-axis, and include units for your answer. **DO NOT EVALUATE THIS INTEGRAL.**

**Include units.**

11) (7 points) Evaluate the integral $\int_1^2 x^2 \ln x \, dx$.

12) (6 points) Find the indefinite integral $\int \frac{x-7}{(x+1)(x-3)} \, dx$.

13) (3 points each) Let $f(x) = \dfrac{1}{x^{2/3}}$.

a) If $a > 0$, then what is the value of $\displaystyle\int_a^8 f(x)\,dx$? (Of course, $a$ will appear in your answer.)

b) Referring to the same $f(x)$ as above, note that $f(x)$ is undefined when $x = 0$. Nonetheless, we still define the improper integral $\displaystyle\int_0^8 f(x)\,dx$.

Give the definition of this improper integral, and use the definition, together with part a), to calculate $\displaystyle\int_0^8 f(x)\,dx$.