Practice Final 1125

1.

$$\left(x^2+1\right)\frac{dy}{dx}=2xy^2,\ y(1)=2$$

$$y^{-2}dy=\frac{2x}{x^2+1}dx$$

$$-\frac{1}{y}=\ln\left(x^2+1\right)+C_1$$

$$y=\frac{1}{C-\ln\left(x^2+1\right)},\ 2=\frac{1}{C-\ln(2)}\ \Rightarrow\ C=\frac{1}{2}+\ln 2$$

$$y=\frac{1}{\frac{1}{2}+\ln 2-\ln\left(x^2+1\right)}$$

$$y=\frac{2}{1+2\ln 2-2\ln\left(x^2+1\right)}$$

2.

$$m\frac{dv}{dt}=mg-kv,\ v(0)=0$$

$$3\frac{dv}{dt}=3(9.8)-6v$$

$$\frac{dv}{dt}=9.8-2v=-2(v-4.9)$$

$$\frac{dv}{v-4.9}=-2\,dt$$

$$\ln\left|v-4.9\right|=-2t+C_1$$

$$v-4.9=Ce^{-2t},\ v(0)=0\ \Rightarrow\ C=-4.9$$

$$v(t)=4.9\left(1-e^{-2t}\right)$$

$$v(10)=4.8999999\ m/s$$

3. $\dfrac{dH}{dt}=50-0.00001H,\ H(0)=500{,}000$

4.

$$y(0)=1$$

$$y(0.1)\approx 1+(1)(0.1)=1.100$$

$$y(0.2)\approx 1.100+(1.220)(0.1)=1.222$$

5. (a) $P=1000$ is an equilibrium solution, the rate of change is zero. In 5 years $P=1000$.

(b) If $P=5000$, $\dfrac{dP}{dt}>0$. The solution curve is increasing to the equilibrium of 10,000.

6. (a) Converges to $\frac{10}{3}$

$$S_n = 2 + \frac{2^2}{5^1} + \frac{2^3}{5^2} + \frac{2^4}{5^3} + \cdots + \frac{2^n}{5^{n-1}}$$

$$\frac{2}{5}S_n = \quad \frac{2^2}{5^1} + \frac{2^3}{5^2} + \frac{2^4}{5^3} + \cdots + \frac{2^n}{5^{n-1}} + \frac{2^{n+1}}{5^n}$$

$$\frac{3}{5}S_n = 2 - \frac{2^{n+1}}{5^n}$$

$$S_n = \frac{5}{3}\left(2 - \frac{2^{n+1}}{5^n}\right)$$

$$S = \lim_{n \to \infty} \frac{5}{3}\left(2 - \frac{2^{n+1}}{5^n}\right) = \frac{10}{3}$$

(b) Converges to $\frac{1}{3}$

$$S_1 = \frac{1}{3} - \frac{1}{4}$$

$$S_2 = \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} = \frac{1}{3} - \frac{1}{5}$$

$$S_3 = \frac{1}{3} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} = \frac{1}{3} - \frac{1}{7}$$

$$S_n = \frac{1}{3} - \frac{1}{n+3}$$

$$S = \lim_{h \to 0} \left(\frac{1}{3} - \frac{1}{n+3}\right) = \frac{1}{3}$$

7. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for $-\infty < x < \infty$, so $\sum_{n=0}^{\infty} \frac{3^n}{n!} = e^3$

8. (a) $\frac{1}{1+x^2}$ is the sum of a geometric series with first term 1 and ratio $-x^2$.

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots, \quad -1 < -x^2 < 1 \Rightarrow -1 < x < 1$$

(b) $\tan^{-1}(x) + C = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$, let $x = 0 \Rightarrow C = 0$

$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, -1 \le x \le 1$ Same center and radius as in part(a).
Convergence at the endpoints because the resulting series are both conv. alt. series.

9. (a) $(1+5x)^{1/5} = 1 + \frac{1}{5}(5x) + \frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)}{2}(5x)^2 + \frac{\left(\frac{1}{5}\right)\left(-\frac{4}{5}\right)\left(-\frac{9}{5}\right)}{2\cdot 3}(5x)^3 = 1 + x - 2x^2 + 6x^3 + \cdots$

interval of conv. $-\frac{1}{5} < x < \frac{1}{5}$.

(b) $c = -2$ and $m = 2$

(c) You must be near the center of the interval of convergence to make this type of statement and $x = 0$ is the center of the interval of convergence.