MATH 1241 - Fall 2009 - Midterm - 10/22 **Total points: 100**

Show all your work for credit.

Name:__________________________

1. (9 pts) The quantity of a drug (in mg) in the body t minutes after it has been administered is given by $Q(t)$ for some function. Suppose we know that at 30 minutes after the drug has been administered, there are 85 mg of the drug in the body. Also from the 30 minutes to the 31 minutes after the drug has been administered, the quantity of the drug in the body decrease by roughly 0.02 mg. Use the information to estimate $Q(30.05)$.

$Q(30.05) \simeq Q(30) + (30.05-30)\cdot Q'(30)$

$= 85 + (0.05)\cdot(-0.02) = 84.999$

2. (9 pts) Find the equation of the line that passes through $f(x) = x^3 - 4x + 8$ at $x = 2$ and is perpendicular to the tangent line of $f(x)$ at $x = 2$. the line has slope $-\frac{1}{f'(2)} = -\frac{1}{8}$ $\left(\begin{array}{l} f'(x) = 3x^2 - 4 \\ f'(2) = 8 \end{array}\right)$

and passes through $(2, f(2)) = (2, 8)$

Hence the eq. $y - 8 = -\frac{1}{8}(x-2)$ $\left(\text{or } y = -\frac{1}{8}x + \frac{33}{4}\right)$

3. (7 pts each) Find the derivatives of the following functions.

(a) $y = \ln(3x^2 + \sin(2x))$

$y' = \frac{1}{3x^2 + \sin 2x}\cdot(6x + 2\cos 2x)$

(b) $y = 5x^3 + 3x^2 - 2x - 5$.

$y' = 15x^2 + 6x - 2$

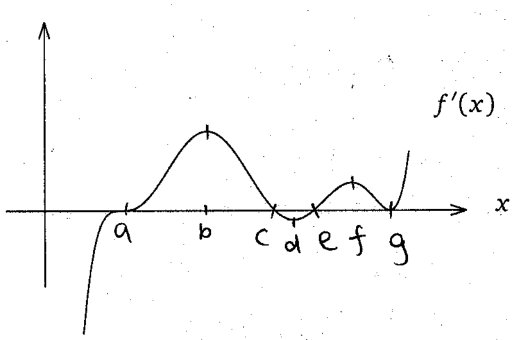
(c) $y = e^{x(x+5)}(32^x + 49)$. $= e^{x^2+5x}\cdot(32^x + 49)$

$y' = e^{x^2+5x}\cdot(2x+5)\cdot(32^x+49) + e^{x^2+5x}\cdot(\ln 32\cdot 32^x)$

(d) $y = 10(3x^2 + 2e^{3x^4})^8$.

$y' = 80(3x^2 + 2e^{3x^4})^7\cdot(6x + 24x^3 e^{3x^4})$

4. The following graph is a graph of a derivative function f'.



(a) (8 pts) Indicate the critical points of the function f. Identify each critical point as a local maximum, a local minimum, or neither.

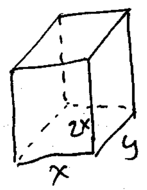
a, c, e, g

local min, local Max, local min, neither

(b) (10 pts) Indicate the inflection points of the function f.

b, d, f, g

5. (10 pts) An open-top cardboard box is to be constructed so that its height is twice its width and its volume is 50 ft^3. What dimensions minimize the cardboard used?



$2x \cdot x \cdot y = 50 \Rightarrow y = 25x^{-2}$

Cardboard used $C(x) = x \cdot 25x^{-2} + 2 \cdot x \cdot 2x + 2 \cdot 2x \cdot 25x^{-2}$

$= 4x^2 + 125x^{-1}$

$C'(x) = 8x - 125x^{-2}$, $C'(x) = 0 \Rightarrow 8x^3 = 125 \Rightarrow x = \frac{5}{2}$

Hence the dimensions is $\frac{5}{2}$ ft $\times$ 5 ft $\times$ 4 ft.

6. Consider the function $f(x) = x^4 - 18x^2 + 25$.

(a) (10 pts) Find the critical points (give the x, and y coordinates). Identify each critical point as a local maximum, a local minimum, or neither.

$f'(x) = 4x^3 - 36x$, $f'(x) = 0 \Rightarrow 4x(x^2 - 9) = 0 \Rightarrow x = -3, 0, 3$

$f''(x) = 12x^2 - 36$, $f''(-3) = 72 > 0$, $f''(0) = -36 < 0$, $f''(3) = 72 > 0$.

Hence $(-3, f(-3)) = (-3, -56)$ and $(3, f(3)) = (3, -56)$ are local min

$(0, f(0)) = (0, 25)$ is the local Max.

(b) (10 pts) Find the inflection points (give the x, and y coordinates).

$f''(x) = 0 \Rightarrow 12(x^2 - 3) = 0 \Rightarrow x = -\sqrt{3}, \sqrt{3}$ $\quad (\sqrt{3} \approx 1.73205\cdots)$

$\left.\begin{array}{l} f''(-2) > 0 \\ f''(0) < 0 \end{array}\right\} \Rightarrow (-\sqrt{3}, f(-\sqrt{3})) = (-\sqrt{3}, -20)$

$\left.\begin{array}{l} f''(0) < 0 \\ f''(2) > 0 \end{array}\right\} \Rightarrow (\sqrt{3}, f(\sqrt{3})) = (\sqrt{3}, -20)$

are inflection pts.

(c) (6 pts) Sketch the graph by using the information about its first and second derivative. Label all the local min., Max. and inflection points.