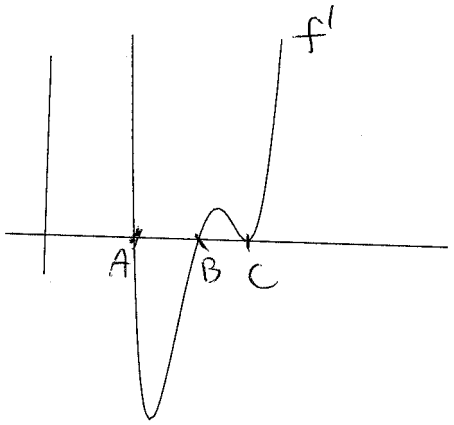
MATH 1241 - Fall 2009 - Quiz 3 - 10/15

Total points: 20+2

Name:

1. (3 pts) The graph on the right is a graph of a derivative function f'. Indicate on a sketch the x-values that are critical points of the function f itself. Identify each critical point as a local maximum, a local minimum, or neither.



A — local Max.

B — local min

C — neither

$f'(A)=0$, f' is decreasing near $x=A$

$\Rightarrow f''(A)<0$

$\Rightarrow f$ is concave down near $x=A$

$\Rightarrow f$ has local Max at $x=A$

2. Find the derivative function for the following functions.

(a) (3 pts) $y = \ln(2x^2)\sin(2x^3)$

$y' = \frac{1}{2x^2}\cdot 4x\cdot \sin(2x^3) + \ln 2x^2 \cdot \cos 2x^3 \cdot 6x^2$

$= 2x^{-1}\cdot \sin(2x^3) + 6x^2 \cdot \ln 2x^2 \cdot \cos(2x^3)$

(b) (3 pts) $y = 5\sqrt[2]{x^5} - \cos(4x) + \sin(8) = 5x^{\frac{5}{2}} - \cos(4x) + \sin(8)$

$y' = 5\cdot\frac{5}{2}x^{\frac{3}{2}} + \sin(4x)\cdot 4$

$= \frac{25}{2}x^{\frac{3}{2}} + 4\sin(4x)$

3. (6 pts) Find the critical points of the function: $f(x) = x^3 + 3x^2 - 45x + 98$. Identify each critical point as a local maximum, a local minimum, or neither.

$f'(x) = 3x^2 + 6x - 45 = 3(x^2 + 2x - 15) = 3(x-3)(x+5)$

$f'(x) = 0 \Rightarrow x = 3, -5$

$f''(x) = 6x + 6 \Rightarrow f''(3) > 0$ and $f''(-5) < 0$

Hence the critical pts are $(3, 17)$ and $(-5, 273)$

$f(3)$ $f(-5)$

and $(3, 17)$ is the local min

$(-5, 273)$ is the local Max.

4. (3 pts) There are 30 apple trees in an orchard. Each tree produces 400 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of ~~trees~~ apples?

Suppose there are x apple trees to be added.

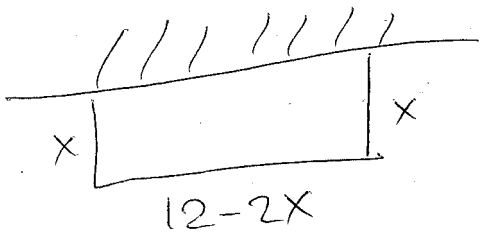
Then the total # of apples are.

$$A(x) = (30+x)(400-10x) = -10x^2+100x+12000$$

$$A'(x) = -20x+100 \quad , A'(x)=0 \Rightarrow x=5 \quad (\text{Also } A''(x)=-20)$$

Hence 5 trees should be added to maximize the total output of apples.

5. (4 pts) A farmer with 12 feet of fence wants to enclose on three sides the largest possible rectangular pen against one wall of his barn. What is the maximum area he can enclose and what are its dimensions?



the area of the rectangular pen.

$$A(x) = x\cdot(12-2x) = 12x-2x^2$$

$$A'(x) = 12-4x \quad , A'(x)=0 \Rightarrow x=3 \quad (\text{Also } A''(x)=-4)$$
$$\Rightarrow A''(3)=-4\ (<0)$$

Hence the dimensions should be

$$3 \times (12-2\cdot 3)$$

$$\boxed{3\text{ft} \times 6\text{ft}.}$$