

## REMEMBERING I.M.GELFAND

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On December 6, 2009, two months after I.M.Gelfand passed away, Rutgers University, his last place of work, held a Gelfand Memorial. This event brought together the crème de la crème of several generations of Moscow mathematicians (now scattered all around the world) whose life in mathematics was to a large extent shaped by I.M.'s influence. Despite the sad occasion, it was a pleasure to see many old friends and to share memories of our student years when we all attended the famous Gelfand Seminar at Moscow State University. And of course to share stories about I.M. He was such a huge presence in so many lives, and his passing left a gap which will be impossible to fill.

I first met I.M. in the early Fall of 1970. The meeting was arranged by Victor Gutenmacher, who at the time worked at the School by Correspondence organized by I.M. (with the purpose of bringing mathematics into the lives of school children all around the Soviet Union). I.M., who as usual was simultaneously involved in a myriad of various projects, both scientific and pedagogical, had decided to organize and run a special mathematical class of seventh-graders within the famous Moscow School No. 2, which already had a decade long tradition of such classes. To help him run this program, I.M. asked Victor to find him several young assistants who had themselves passed through such a class during their school years. I was very fortunate to become one of four such assistants. (Another was my old friend and classmate since the seventh grade Borya Feigin, who is now a distinguished mathematician.) All four of us had graduated a year earlier from the same School No. 2, all were math undergraduates beginning our second year at Moscow State University, and all felt a little lost in our mathematical studies.

The organizational matters could have been resolved within a few minutes, but our first meeting with I.M. lasted for several hours. The four of us walked with I.M. for hours, and he talked to us about all kinds of things, mathematical and not. He asked us what we most loved about mathematics and what seminars and elective courses we had attended during our freshman year. Of course, he declared that we had done everything wrong and were almost lost for mathematics, but there was still some little hope for us if we started to attend his seminar at once. He explained to us how to study a new mathematical subject – focus on the most basic things at the foundation, and dwell upon them until you reach full understanding; then the technicalities of the subject would be understood very quickly and effortlessly. I remember vividly how he illustrated this by explaining to us the foundations of linear algebra: a subspace in a vector space is characterized by one integer, its dimension; a pair of subspaces is characterized by three integers (dimensions of the two subspaces

and of their intersection); what about a triple of subspaces? and what about a quadruple? <sup>1</sup>

This meeting was definitely one of my most life-changing experiences. I have never met a person with such personal magnetism and such an ability to ignite enthusiasm about mathematics as I.M. I remember returning home late in the evening, totally exhausted but happy, with the definite feeling that my mathematical fate was sealed that day. I could not sleep and spent most of the night thinking about mysteries of triples of subspaces!

I attended Gelfand Seminar for almost 20 years. A lot has been said about its unique character, including horror stories about I.M.'s rough treatment of speakers and participants. I have had my share of humbling experiences there, both as a speaker and as a "control listener" who was sent by I.M. to the blackboard in the middle of a talk to explain what the speaker was trying to say. Many people, including some excellent mathematicians, could not stand this style and stopped attending the seminar. Those who stayed (myself included) decided that such a great learning experience was worth a little suffering. Equally important, or maybe even more important than the talks themselves was Gelfand's choice of topics, his comments and monologues, which often deviated a lot from the original topic, and of course his famous "pedagogical" jokes and stories.

The official starting time of the seminar was 7pm on Mondays, but it almost always started with much delay, sometimes after up to two hours! I am sure I.M. did this on purpose, because these weekly get-togethers before the seminar with numerous friends coming to the University from all around the city were also a big attraction. Sometimes even after his arrival I.M. did not immediately start the seminar but stayed in the corridor for some time and chatted with people just like everybody else.

I think of myself as I.M.'s "mathematical grandson": my first real teacher and de facto Ph.D. advisor was Joseph Bernstein, one of Gelfand's best students. <sup>2</sup> However, at some point after Joseph's emigration (in 1981, I believe), I.M. approached me and suggested that we start working together. Our close mathematical collaboration lasted about a decade (from the first joint note in 1984 to our 1994 book with Misha Kapranov). It would take volumes to give a comprehensive account of I.M.'s mathematical contributions, so let me just share my personal impressions of some of his unique features as a mathematician and a teacher. <sup>3</sup>

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<sup>1</sup>Triples of subspaces demonstrate the limitations of a fruitful analogy between subspaces of a finite-dimensional vector space and subsets of a finite set: the subspace lattice is only modular but not distributive. Thinking about this analogy led me to my first two published notes that appeared within the next couple of years. As for the quadruples, as I realized much later, I.M. had just finished his remarkable paper with V. A. Ponomarev "Problems of linear algebra and classification of quadruples of subspaces in a finite-dimensional vector space," so he was talking to us about his own cutting-edge research!

<sup>2</sup>Joseph could not be my official advisor since he was never affiliated with the mathematics department at Moscow State, which was also the case for many other first rate mathematicians from the "Gelfand circle." A .A. Kirillov, Sr., also a student of Gelfand, kindly agreed to serve as my official advisor; of course, I also learned a lot from him and from attending his seminar.

<sup>3</sup>For I.M. these two occupations were inseparable. His way of doing mathematics always involved close personal interaction with his innumerable students and collaborators.

Working with him could be a very frustrating experience. Just looking at his incredibly prolific scientific output, one would imagine him as a model of efficiency, never wasting a moment of his time. But most of the time during our daily meetings was filled with numerous distractions, jumps from one topic to another, his long phone conversations with an amazing variety of people on an amazing variety of topics, etc. Quite often after long hours spent like this I felt totally exhausted and had a depressing feeling that we had just wasted a perfectly good working day. But almost without exception there came a moment (sometimes when I was already saying good-bye at the door) of his total concentration on our project, which led to an extremely rapid progress completely justifying all the torturous hours leading to this moment. It seemed as if his subconscious mind never stopped working on our project (and probably on a multitude of other things at the same time), and it just took I.M. a long time to become ready to spell out the results of this work.

This “non-linearity” of I.M.’s thinking process was also one of the many features that made his seminar so unique. He would spend an inordinate amount of time asking everybody to explain to him some basic definitions and facts, and just when most of the participants (starting with the speaker, of course) would get totally frustrated, I.M. would suddenly switch gears and say something very illuminating, making it all worthwhile.<sup>4</sup>

I have never met any other mathematician with such an ability to see the “big picture” and always go to the heart of the matter, ignoring unnecessary technicalities. He had an uncanny ability to ask the “right” questions, and to find unexpected connections between different mathematical fields. I.M. was fully aware of this gift and liked to illustrate it by one of his numerous “pedagogical” stories: “An old plumber comes to repair a heater. He goes around it, thinks for a moment, and hits it once with a hammer. The heater immediately starts working. The plumber charges 200 rubles for his service. The owner says: ”But you just spent two minutes and didn’t do anything.” The plumber replies: ”I am charging you 3 rubles for hitting your heater with a hammer, and the rest for knowing where to hit, which took me forty years to learn!”<sup>5</sup>

I.M. had such an enormous wealth of ideas and plans that he needed many collaborators to help him realize even a small part of it. I was always very impressed with his great intuitive grasp of people. A few minutes of penetrating questioning of a new acquaintance allowed him to take the full measure of a person, understand his or her scientific potential, strengths and weaknesses, even how much pressure the person could withstand. In my own case, it seems that I.M. detected my soft spot for algebraic combinatorics (quite rare in Moscow at that time, I must say) even before I realized it myself. Very soon after I joined his seminar, he asked me to study (and then give a talk on) the old book “The Theory of Group Characters

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<sup>4</sup>I.M. often interrupted speakers with the words “May I ask a stupid question?” The best reply to this was given by Y .I. Manin during one of his rare appearances as a speaker: “No, I.M., I don’t think you are capable of such a thing!”

<sup>5</sup>I.M.’s amazing record of initiating new fruitful directions of mathematical research provides plenty of examples of “knowing where to hit.” Let me just mention one example: the theory of general hypergeometric functions initiated by I.M. (where a significant part of my collaboration with him took place) grew from his insight that hypergeometric functions should live on the Grassmannians.

and Matrix Representations of Groups” by D.E. Littlewood, where the representation theory was treated with a strong combinatorial flavor. This was truly a sniper’s shot: the facts and ideas that I learned from this book continue to serve me to this day. Another truly inspired suggestion by I.M. was to bring together Borya Feigin and Dmitry Borisovich Fuchs, which led to many years of very fruitful collaboration.

Like so many of my friends and colleagues, I feel very fortunate for knowing Israel Moiseevich and being given a chance to be close to this gigantic, complex, wise, inspiring and infinitely fascinating personality.

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