Math U241, Fall 2007, Quiz #10

Name: Solutions

Problem 1 Use Newton's method to estimate $\sqrt[4]{80}$ correct to 8 decimal places using the initial approximation $x_1 = 3$. Indicate to which function you apply Newton's method, what is the formula for x_{n+1} in terms of x_n, and what are the numbers $x_2, x_3, \ldots$.

$f(x) = x^4 - 80$

$f'(x) = 4x^3$

$$x_{n+1} = x_n - \frac{x_n^4 - 80}{4x_n^3}$$

$x_1 = 3$

$x_2 = 2,99074074$

$x_3 = 2.99069756$

$\boxed{x_4 = 2.99069756}$

Problem 2: Find the antiderivative $F(x)$ of the function $f(x) = x^{4/3} + x^{3/4}$, satisfying the condition $F(1) = 0$.

$$F(x) = \frac{3}{7}x^{7/3} + \frac{4}{7}x^{7/4} + C$$

$$0 = F(1) = \frac{3}{7} + \frac{4}{7} + C = 1 + C.$$

$C = -1.$

$$\boxed{F(x) = \frac{3}{7}x^{7/3} + \frac{4}{7}x^{7/4} - 1}$$

Problem 3: A particle moves along a straight line with acceleration function $a(t) = 3e^t - 2\sin t$. Find the velocity function $v(t)$ if the initial velocity is $v(0) = 0$.

$v'(t) = a(t) = 3e^t - 2\sin t$

$v(t) = 3e^t + 2\cos t + C$

$$0 = v(0) = 3\underbrace{e^0}_{1} + 2\underbrace{\cos 0}_{1} + C = 5 + C.$$

$C = -5$

Answer: $\boxed{v(t) = 3e^t + 2\cos t - 5}$