

Math U241, Fall 2007, Quiz #2

Name: Solutions

Problem 1 (a) Compute the derivative of the function $f(x) = \frac{1}{x+1}$ using the definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+1+h} - \frac{1}{x+1}}{h} = \lim_{h \rightarrow 0} \frac{x+1 - (x+1+h)}{h(x+1+h)(x+1)} =$$

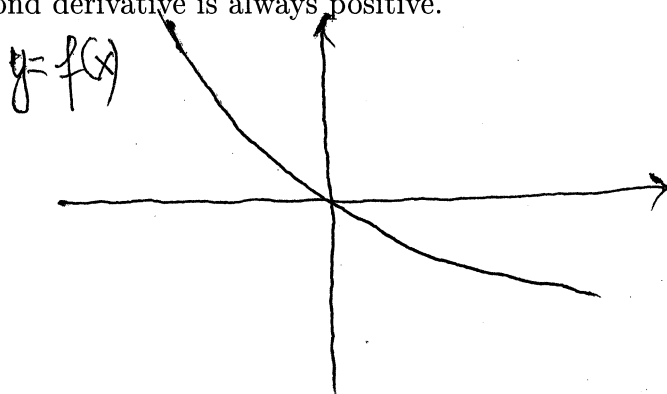
$$\lim_{h \rightarrow 0} \frac{-h}{h(x+1+h)(x+1)} = \lim_{h \rightarrow 0} \frac{-1}{(x+1+h)(x+1)} = -\frac{1}{(x+1)^2}$$

(b) Find the equation of the tangent line to the graph of $y = \frac{1}{x+1}$ at the point $(0, 1)$.

$$f(0) = 1, \quad f'(0) = -1.$$

Answer: $y = 1 - x$

Problem 2: Sketch the graph of a function whose first derivative is always negative and whose second derivative is always positive.



$f' < 0$ so f decreases
 $f'' > 0$ so f concave upward

Problem 3: Suppose that the derivative of a function $f(x)$ is given by $f'(x) = 3x - x^2$. Sketch the graph of $f(x)$ and find the intervals on which f is increasing or decreasing.

