Name: Solutions
(PLEASE PRINT)

MTH U230 Discrete Mathematics, Fall 2007, Quiz 3

1) Show the Boolean identity $(x+y)(x+z)(y+z) = xy + xz + yz$.

$$\underbrace{(x+y)(x+z)}_{x+yz}(y+z) = (x+yz)(y+z) = xy + \overset{y}{yy}z + xz + y\overset{z}{zz} =$$

$$xy + xz + \underbrace{yz + yz}_{yz} = xy + xz + yz.$$

2) Find the sum-of-products expansion of the Boolean function $(x+y+z)(\bar{x}+\bar{y}+\bar{z})$.

$$(x+y+z)\overline{\ } = x\bar{y} + x\bar{z} + y\bar{x} + y\bar{z} + z\bar{x} + z\bar{y} \quad (\text{Since } x\bar{x} = y\bar{y} = z\bar{z} = 0)$$

$$= x\bar{y}(z+\bar{z}) + x(y+\bar{y})\bar{z} + \bar{x}y(z+\bar{z}) + (x+\bar{x})y\bar{z} + \bar{x}(y+\bar{y})z +$$

$$(x+\bar{x})\bar{y}z = x\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z$$

3) Find the sum-of-products expansion of the Boolean function $F(x,y,z)$ that equals 1 if and only if $y+z=0$.

$$F(x,y,z) = \overline{y+z} = \bar{y}\bar{z} = (x+\bar{x})\bar{y}\bar{z} = x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}$$

4) How many different Boolean functions $F(x,y)$ are there such that $F(\bar{x},\bar{y}) = F(x,y)$ for all values of the Boolean variables x and y?

Conditions: $F(0,0) = F(1,1)$, $F(1,0) = F(0,1)$.

So F is determined by its values $F(1,1)$ and $F(0,1)$, which can be chosen arbitrarily. Answer: 4 different functions.