Math U241, Fall 2007, Quiz #5

Name: Solutions

1. Find the derivative of the function $y = 10^{\sin(x^2)}$.

$u = \sin(x^2)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 10^{\sin(x^2)} \cdot \ln 10 \cdot (\sin(x^2))' =$$

$$10^{\sin(x^2)} \cdot \ln 10 \cdot \cos(x^2) \cdot 2x = \boxed{2 \cdot \ln 10 \cdot 10^{\sin(x^2)} \cdot \cos(x^2)}$$

2. Find dy/dx by the implicit differentiation of the equation $y\sin(x^2) = x\sin(y^2)$.

$$\frac{dy}{dx} \cdot \sin(x^2) + y \cdot \cos(x^2) \cdot 2x = \sin(y^2) + x \cdot \cos(y^2) \cdot 2y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\sin(x^2) - 2xy\cos(y^2)\right) = \sin(y^2) - 2xy\cos(x^2)$$

$$\boxed{\frac{dy}{dx} = \frac{\sin(y^2) - 2xy\cos(x^2)}{\sin(x^2) - 2xy\cos(y^2)}}$$

3. Check that the point $(8, 1)$ lies on the curve given by $x^{2/3} + y^{2/3} = 5$, and find an equation of the tangent line to the curve at this point.

$8^{2/3} + 1^{2/3} = 4 + 1 = 5.$ ✓

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} = 0$$

Plug in $x=8, y=1$

$8^{-1/3} = \frac{1}{2}$

$1^{-1/3} = 1$

$$\frac{2}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1}{2}$$

Tangent line:

$$\frac{y-1}{x-8} = -\frac{1}{2}$$

$$2y - 2 = 8 - x$$

$$\boxed{y = -\frac{1}{2}x + 5}$$