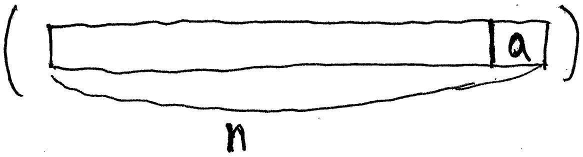
Name: Solutions
(PLEASE PRINT)

MTH U230 Discrete Mathematics, Fall 2007, Quiz 6

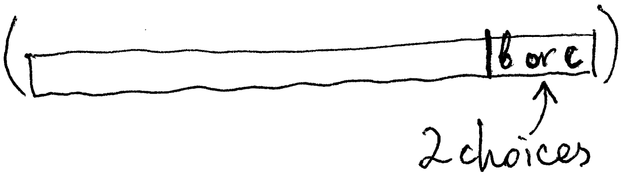
1) Find the solution of the recurrence relation $a_n = 5a_{n-1} - 3$ with the initial condition $a_0 = 1$.

$$a_n = 5a_{n-1} - 3 = 5(5a_{n-2} - 3) - 3 = 5^2 a_{n-2} - 5\cdot 3 - 3 =$$
$$5^2(5a_{n-3} - 3) - 5\cdot 3 - 3 = 5^3 a_{n-3} - 5^2\cdot 3 - 5\cdot 3 - 3 = \dots =$$
$$5^n a_0 - 5^{n-1}\cdot 3 - 5^{n-2}\cdot 3 - \dots - 5\cdot 3 - 3 = 5^n - 3\cdot\frac{5^n - 1}{4} =$$
$$\boxed{\frac{5^n + 3}{4}}$$

2) Find a recurrence relation for the number of strings of length n consisting of letters a, b and c, and containing an even number of a's. ← s_n



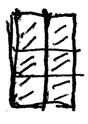
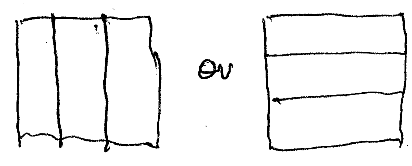
$\# (\;\dots\; a) = 3^{n-1} - s_{n-1}$

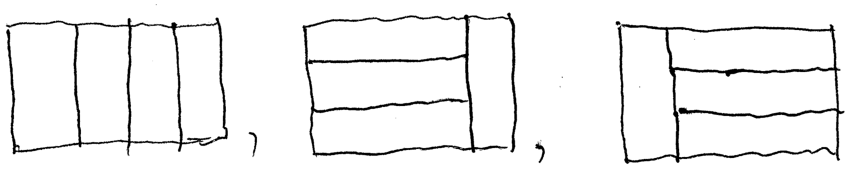


$\# (\;\dots\; b \text{ or } c) = 2s_{n-1}$

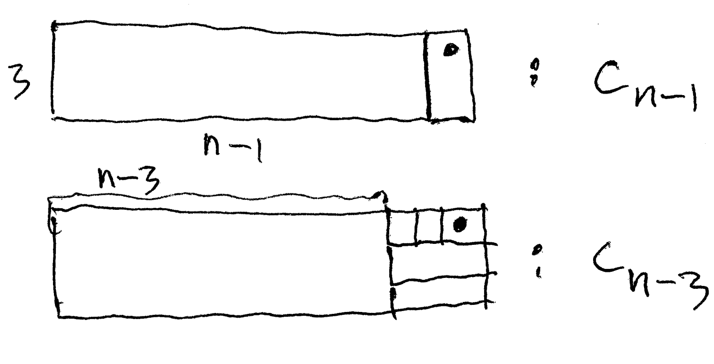
So: $s_n = \boxed{(3^{n-1} - s_{n-1}) + 2s_{n-1}} = \boxed{3^{n-1} + s_{n-1}}$

3) Let c_n be the number of ways a $3 \times n$ rectangular board can be tiled using 1×3 pieces.
(a) Compute c_1, c_2, c_3 and c_4.

$c_1 = 1$:  $c_2 = 1$:  $c_3 = 2$: 

$c_4 = 3$: 

(b) Find a recurrence relation for (c_n).



$$c_n = c_{n-1} + c_{n-3}$$
$$(n \geq 4).$$