Math U241, Fall 2007, Quiz #6

Name: Solutions

Find the derivatives of the following functions:

1. $y = \ln(e^x \sin x + e^{-x} \cos x)$.

$$y' = \frac{1}{e^x \sin x + e^{-x}\cos x} \cdot (e^x \sin x + e^{-x}\cos x)' =$$

$$\frac{e^x \sin x + e^x \cos x - e^{-x}\cos x - e^{-x}\sin x}{e^x \sin x + e^{-x}\cos x} = \frac{(e^x - e^{-x})(\sin x + \cos x)}{e^x \sin x + e^{-x}\cos x}$$

2. $y = \sin(x^2)^{\ln x}$.

$$\ln y = \ln x \cdot \ln(\sin(x^2))$$

$$\frac{1}{y} \cdot y' = \frac{1}{x}\ln(\sin(x^2)) + \frac{\ln x}{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$$

$$y' = \sin(x^2)^{\ln x} \cdot \left(\frac{\ln(\sin(x^2))}{x} + \frac{2x \cdot \ln x \cdot \cos(x^2)}{\sin(x^2)}\right)$$

3. $y = \dfrac{5^x \tan^{-1} x}{(2x+1)^{10}}$.

$$\ln y = x\ln 5 + \ln(\tan^{-1} x) - 10\ln(2x+1)$$

$$\frac{1}{y} \cdot y' = \ln 5 + \frac{1}{(\tan^{-1} x)\cdot(x^2+1)} - \frac{10 \cdot 2}{2x+1}$$

$$y' = \frac{5^x \cdot \tan^{-1} x}{(2x+1)^{10}}\left(\ln 5 + \frac{1}{(x^2+1)\tan^{-1} x} - \frac{20}{2x+1}\right)$$