Math U241, Fall 2007, Quiz #7

Name: Solutions

Problem 1 Find the linear approximation of the function $f(x) = x^{4/3}$ at $a = 8$, and use it to approximate $8.1^{4/3}$.

$f(a) = 8^{4/3} = 16.$ $f'(x) = \frac{4}{3}x^{1/3}.$ $f'(a) = \frac{4}{3} \cdot 8^{1/3} = \frac{8}{3}$

$L(x) = 16 + \frac{8}{3}(x-8) = \underbrace{\frac{8}{3}x - \frac{16}{3}}$

$8.1^{4/3} = f(8.1) \approx L(8.1) = \cancel{8}\, 16 + \frac{8}{3} \cdot 0.1 = \frac{244}{15} \approx 16.27$

Problem 2: The volume of a cube was measured 64 cm^3 with a possible error of 4.8 cm^3. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing the edge of the cube.

x = volume of the cube

y = edge of the cube.

$y = x^{1/3}$ $y' = \frac{1}{3} \cdot x^{-2/3}$

$dy = y'dx = \frac{1}{3} \cdot \underbrace{x}_{64}^{-2/3} \cdot \underbrace{dx}_{4.8}$

Max. error: $\frac{1}{3} \cdot 64^{-2/3} \cdot 4.8 = \boxed{0.1}$

Relative error: $dy/y = 0.1/64^{1/3} = 0.1/4 = \boxed{0.025}$

Percentage error: $\boxed{2.5\%}$

Problem 3: The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 4 cm/s. At what rate is the area of the rectangle increasing when the length is 20 cm and the width is 10 cm?

$l(t)$: length of a rectangle at a moment t

$w(t)$: width ——"——

$y(t)$: aree ——"——

$y = l \cdot w$, so $\frac{dy}{dt} = \underbrace{\frac{dl}{dt}}_{8} \cdot \overset{10}{w} + \overset{20}{l} \cdot \underbrace{\frac{dw}{dt}}_{4} = 8 \cdot 10 + 20 \cdot 4 = \boxed{160}$