Name: Solutions

(PLEASE PRINT)

**MTH U165 Intro to Mathematical Reasoning, Fall 2007, Quiz 8**

1) Find a polynomial formula for $a_n = 0^3 + 1^3 + 2^3 + \cdots + n^3$.

$(\Delta a)_n = a_{n+1} - a_n = (n+1)^3$ : polynomial of degree 3.

So $a_n$ = polynomial in $n$ of degree 4.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $a$: | 0 | $a_1$ | $a_2$ | $a_3$ | ... | | | | |
| $\Delta a$: | | 1 | 8 | 27 | 64 | 125 | ... | | |
| $\Delta^2 a$: | | | 7 | 19 | 37 | 61 | ... | | |
| $\Delta^3 a$: | | | | 12 | 18 | 24 | ... | | |
| $\Delta^4 a$: | | | | | 6 | 6 | ... | | |
| $\Delta^5 a$: | | | | | | 0 | 0 | ... | |

$$\boxed{a_n = \binom{n}{1} + 7\binom{n}{2} + 12\binom{n}{3} + 6\binom{n}{4}}$$

2) Find the sequence given by a polynomial expression of degree 3 and such that $a_n = 3^n$ for $n = 0, 1, 2, 3$.

| | | | | | |
|---|---|---|---|---|---|
| $a$: | 1 | 3 | 9 | 27 | |
| $\Delta a$: | | 2 | 6 | 18 | |
| $\Delta^2 a$: | | | 4 | 12 | |
| $\Delta^3 a$: | | | | 8 | |
| $\Delta^4 a$: | | | | 0 | 0 ... |

$$\boxed{a_n = 1 + 2\binom{n}{1} + 4\binom{n}{2} + 8\binom{n}{3}}$$

3) Compute $(\Delta^{10} a)_n$ for the sequence $a = (a_0, a_1, a_2, \ldots)$ given by $a_n = 3^n$ for all $n$.

$(\Delta a)_n = 3^{n+1} - 3^n = 3^n \cdot 3 - 3^n \cdot 1 = 2 \cdot 3^n$

So $(\Delta^2 a)_n = 2^2 \cdot 3^n$, $(\Delta^3 a)_n = 2^3 \cdot 3^n$, ...

$$\boxed{(\Delta^{10} a)_n = 2^{10} \cdot 3^n = 1024 \cdot 3^n}$$