

Name: Solutions
(PLEASE PRINT)

MTH U230 Discrete Mathematics, Fall 2007, Quiz 8

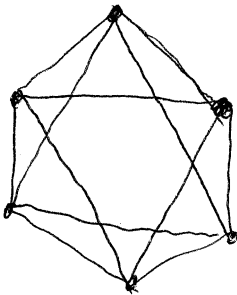
1) How many ways are there to split eight students among four available coop jobs, so that every job will have at least one student assigned to it?

$U = \{\text{all assignments of jobs to students}\} = \{\text{functions: } \{\text{students}\} \rightarrow \{\text{jobs}\}\}$
 $A_i = \{\text{assignments missing job } i\}$ for $i=1,2,3,4$.
 Answer: $|U - (A_1 \cup A_2 \cup A_3 \cup A_4)| = |U| - (|A_1| + \dots + |A_4|) + (|A_1 \cap A_2| + \dots + |A_3 \cap A_4|) - (\text{cardinalities of triple intersections}) + |A_1 \cap A_2 \cap A_3 \cap A_4| =$
 $4^8 - 4 \cdot 3^8 + \binom{4}{2} \cdot 2^8 - \binom{4}{3} \cdot 1 = 4^8 - 4 \cdot 3^8 + 6 \cdot 2^8 - 4$

2) How many permutations of numbers 1, 2, 3, 4, 5, 6, 7, 8 leave exactly three numbers in their original position?

#(choices for 3 numbers to be in their original position) = $\binom{8}{3} =$
 $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$.
 ← derangement number
 Answer: $56 \cdot D_5 = 56 \cdot 5! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$

3) Construct a simple graph with 6 vertices such that every vertex has degree 4.



4) (Bonus problem) Show that for every $n \geq 5$, there exists a simple graph with n vertices and every vertex of degree 4.

Take vertices of a regular n -gon as vertices.
 Edges: sides and shortest possible diagonals (cutting off one vertex).