Math U241, Fall 2007, Quiz #8

Name: Solutions

Problem 1 Find the absolute maximum and absolute minimum of the function $f(x) = x^3 - 6x^2 + 9x + 2$ on the interval $[-1, 2]$.

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

Critical numbers are $x=1$ and $x=3$ but $x=3$ does not belong to $[-1,2]$. So the only critical number in $[-1,2]$ is $x=1$.

endpoints.

x	-1	1	2
$f(x)$	-14	6	4

Absolute max: $f(x) = 6$ at $x=1$.

Absolute min: $f(x) = -14$ at $x=-1$

Problem 2: For the function $g(t) = t^2 e^{-t}$:

(a) Find the intervals on which g is increasing or decreasing, and the local maximum and minimum values of g. (Domain of g: all real numbers).

$g'(t) = 2te^{-t} - t^2e^{-t} = (2t - t^2)e^{-t} = t(2-t)e^{-t}$

Critical numbers: $t=0$, and $t=2$.

$g'(t) > 0$ on $(0,2)$. So g increases on $(0,2)$ and

$g'(t) < 0$ for $t<0$ or $t>2$. decreases for $t<0$ or $t>2$.

$\boxed{t=2: \text{ local max; } g(2) = 4e^{-2} \approx 4/2.7^2}$ $\boxed{t=0: \text{ local min. } g(0) = 0}$

(b) Find the inflection points and the intervals of concavity.

$g''(t) = (2-2t)e^{-t} - (2t - t^2)e^{-t} = (t^2 - 4t + 2)e^{-t}$.

$g''(t) = 0$: $t^2 - 4t + 2 = 0$. $t = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$.

$g''(t) > 0$ for $t > 2 + \sqrt{2} \approx 3.4$, and for $t < 2 - \sqrt{2} \approx 0.6$,

Concave Up.

$g''(t) < 0$ for $\underset{\approx 0.6}{2-\sqrt{2}} < t < \underset{\approx 3.4}{2+\sqrt{2}}$, ← Concave down