Math U241, Fall 2007, Quiz #9

Name: Solutions

Problem 1 A rectangular storage container with square base is to have a volume of 8000 cm^3. Material for the base costs $\$3/cm^2$, material for the sides costs $\$2/cm^2$, and material for the top costs $\$1/cm^2$. Find the dimensions of the cheapest such container.

x cm: side of the Base

h cm: height

Volume: $x^2h = 8000$ so $h = 8000x^{-2}$

Total cost (\$):

$C = 3x^2 + 8xh + x^2$

$= 4x^2 + 8xh = 4(x^2 + 16000x^{-1})$

$C'(x) = 4(2x - 16000x^{-2})$

$C'(x) = 0$: $x = 8000x^{-2}$,

$x^3 = 8000$ $\boxed{x = 20}$

$\boxed{h = \frac{8000}{20^2} = 20}$

$C'(x) > 0$ for $x > 20$

$C'(x) < 0$ for $x < 20$

$\boxed{x = 20,\ h = 20}$ minimum cost.

Problem 2: Find the slope of the shortest line segment that is cut off by the first quadrant and passes through the point (1, 8). Line: $y - 8 = -m(x-1)$ (Slope $= -m$).

x-intercept: $8 = mx - m$, so $x = (8+m)m^{-1}$

y-intercept: $y = 8 + m$. Length: $L = (x^2 + y^2)^{1/2}$

$= ((8+m)^2 m^{-2} + (8+m)^2)^{1/2}$

$= (8+m)(m^{-2}+1)^{1/2}$

$L'(m) = (m^{-2}+1)^{1/2} - m^{-3}(8+m)(m^{-2}+1)^{-1/2} =$

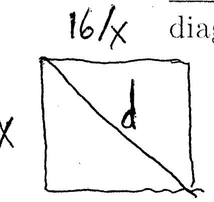
$(m^{-2}+1)^{-1/2}(m^{-2} + 1 - 8m^{-3} - m^{-2}) =$

$\underbrace{(m^{-2}+1)^{-1/2}}_{>0}(1 - 8m^{-3})$. $L'(m) = 0 \Rightarrow$

$1 = 8m^{-3}$, $m^3 = 8$, $\boxed{m = 2}$

Answer: Slope is $\boxed{-2}$

Problem 2: Find the dimensions of a rectangle with area $16 cm^2$ and the shortest possible diagonal.



$D = d^2 = x^2 + \left(\frac{16}{x}\right)^2 = x^2 + 16^2 \cdot x^{-2}$

minimize!

$D' = 2x - 2 \cdot 16^2 x^{-3}$

$= 2x^{-3}(x^4 - 16^2)$

Critical number: $x^4 = 16^2$

$\boxed{x = 4}$ $D' > 0$ for $x > 4$

$\boxed{16/x = 4}$ $D' < 0$ for $x < 4$

Answer: Square 4×4 cm^2.