Titles and Abstracts

Jean-Michel Bismut (U. Paris-Sud, Orsay, France),
Title: *The hypoelliptic Laplacian and its applications*

Jochen Brüning (Humboldt University, Berlin, Germany)
Title: *The signature operator on Riemannian pseudomanifolds*
Abstract: We consider an oriented Riemannian manifold which can be compactified by adjoining a smooth compact oriented Riemannian manifold, B, of codimension at least two, such that a neighbourhood of the singular stratum is given by a family of metric cones. We show that there is a natural self-adjoint extension for the Dirac operator on smooth compactly supported differential forms with discrete spectrum, and we determine the condition of essential self-adjointness. We describe the boundary conditions analytically and construct a good parametrix which leads to the asymptotic expansion of the associated heat trace. We also give a new proof of the local formula for the L^2-signature.

Gregory Eskin (UCLA)
Title: *Artificial Black Holes*
Abstract: Artificial black holes are the black holes for the linear equations describing the wave propagation in a moving medium. They are called artificial black holes (or acoustic or optical black holes) to distinguish from the black holes in the general relativity. We find the conditions for the existence of black holes in the case of two space dimensions and in the case of axisymmetric three-dimensional space. We study the impact of black holes on the nonuniqueness in the inverse hyperbolic problems.

Boris Fedosov (Moscow Institute of Electronic Technology, Russia)
Title: *On a spectral theorem for deformation quantization*
Abstract: We consider a notion of an eigenstate in deformation quantization and introduce an analog of a spectral decomposition for Morse Hamiltonians. In particular, for non-critical values of the Hamiltonian the first three terms of the eigenstate functional are founfd explicitly, the contributions of critical points are investigated and a perturbation theory for such contributions is developed.

Andrei Gabrielov (Purdue University)
Title: *Irreducibility of some spectral determinants*
Abstract: Eigenfunctions of the even quartic oscillator, i.e., Schrodinger operator with an even polynomial potential of degree four, are associated with certain properly embedded infinite planar trees. The braid group action on the trees helps us to understand the dependence of

the eigenfunctions and the corresponding eigenvalues on the coefficients of the potential. In particular, we give a rigorous proof of the fact, discovered by Bender and Wu 40 years ago, that the spectral determinant of the even quartic oscillator has exactly two irreducible components. Similar results are obtained for several other one-parametric families of eigenvalue problems.

(joint work with Alex Eremenko)

Carolyn S. Gordon (Dartmouth College)
Title: *Inverse spectral results for Schrödinger operators on line bundles*
Abstract: Let M be a closed Riemannian manifold M and L a Hermitian line bundle over M. Each Hermitian connection on L gives rise to a Laplace operator acting on sections of L. Given a potential function Q on M, one further obtains a Schrödinger operator on L. We consider the questions: To what extent does the spectrum of the Laplace operator determine the connection or the curvature of the connection? To what extent does the spectrum of the Schrödinger operator determine the connection and the potential? We will review various positive results in the case of flat tori and will discuss negative results for flat tori and Riemann surfaces. We will also consider the Laplacians and Schrödinger operators on all the higher tensor powers of the line bundle in the two settings.

Victor Guillemin (M.I.T.)
Title: *Some inverse spectral results for the semi-classical Schroedinger operator*
Abstract: I'll report in this talk on some recent work, joint with Alex Uribe, on the extent to which one can recapture the potential in the semi-classical Schroedinger operator from spectral data on a small interval about the lowest ground state. (This generalizes earlier work of ours on "symmetric" potential wells.)

Bernard Helffer (U. Paris-Sud, Orsay, France)
Title: *Semiclassical analysis for the ground state energy of a Schroedinger operator with magnetic field*

Victor Ivrii (U.Toronto, Canada)
Title: *Spectral Asymptotics and Dynamics*
Abstract: I am going to show that

- Eigenvalue asymptotics is intimately related to the corresponding dynamics;
- There is a quantum dynamics and classical dynamics and both of them are intimately related;
- Classical dynamics may be branching;
- Quantum dynamics is richer than the classical one;

- Standard notions, in particular periodicity, of the classical dynamics are not that obvious in many situations (due to branching)

Yuri Kordyukov (Institute of Mathematics, Ufa, Russia)
Title: *Adiabatic limits on foliated manifolds*
Abstract: We will describe some old and new results on adiabatic limits on foliated manifolds. Among the topics we are going to discuss, we mention: adiabatic asymptotics for the spectrum distribution function of the Laplace operator, small eigenvalues of the Laplace operator in the adiabatic limit and the differentiable spectral sequence of the foliation, applications of the adiabatic limits to some problems in geometry of foliations and some open problems.

Ari Laptev (KTH, Stockholm and Imperial College London)
Title: *Spectral inequalities for a class of hypo-elliptic operators*

G.L. Litvinov (Independent University of Moscow)
Title: *The Maslov Dequantization, Tropical Mathematics, and Geometry*
Abstract: Tropical mathematics can be treated as a result of a dequantization of the traditional mathematics as the Planck constant tends to zero taking imaginary values, see [1]-[4]. This kind of dequantization is known as the Maslov dequantization and it leads to a mathematics over tropical algebras like the max-plus algebra. The so-called idempotent dequantization is a generalization of the Maslov dequantization. The idempotent dequantization leads to idempotent mathematics over idempotent semirings. For example, the field of real or complex numbers can be treated as a quantum object whereas idempotent semirings can be examined as "classical" or "semiclassical" objects (a semiring is called idempotent if the semiring addition is idempotent, i.e. $x + x = x$.

Tropical mathematics is a part of idempotent mathematics. Tropical algebraic geometry can be treated as a result of the Maslov dequantization applied to the traditional algebraic geometry (O.Viro, G.Mikhalkin). There are interesting relations and applications to the traditional convex geometry [3].

In the spirit of N.Bohr's correspondence principle there is a (heuristic) correspondence between important, useful, and interesting constructions and results over fields and similar results over idempotent semirings. A systematic application of this correspondence principle leads to a variety of theoretical and applied results, see, e.g., [1]-[6].

In the framework of idempotent mathematics, a new version of functional analysis is developed from idempotent variants of basic theorems (e.g., of the Hahn-Banach type) to the theory of tensor products, nuclear operators and nuclear spaces in the spirit of A. Grothendieck as well as basic concepts and results of the theory of representation of groups in idempotent linear spaces, see, e.g., [1]-[4].

Last time the Maslov dequantization and related dequantization procedures are applied to different concrete mathematical objects and structures, see, e.g., [3]-[6].

Examples:

1. The Legendre transform can be treated as a result of the Maslov dequantization of the Fourier-Laplace transform (V.P. Maslov).

2. If f is a polynomial, then a dequantization procedure leads to the Newton polytope of f. Using the so-called dequantization transform it is possible to generalize this result to a wide class of functions and convex sets, see [3].

3. An application of dequantization procedures to linear operators leads to spectral properties of these operators, see [6].

4. An application of a dequantization procedure to metrics leads to the Hausdorff-Besicovich dimension including the fractal dimension, see [6].

5. An application of a dequantization procedure to measures and differential forms leads to a notion of dimension at a point, see [6]. This dimension can be real-valued (e.g., negative).

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References

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the same in: Idempotency/ J. Gunawardena (Ed.), Cambridge University Press, Cambridge, 1998, p. 420443; E-print arXiv:math.GM/0101021.

[2] G.L. Litvinov and V.P. Maslov, eds., Idempotent Mathematics and Mathematical Physics, Contemporary Mathematics, Vol. 377, Amer. Math. Soc., Providence, Rhode Island, 2005 – 370 pp.

[3] G.L. Litvinov and G.B. Shpiz, The dequantization transform and generalized Newton polytopes. – Ibid., 2005, p. 181186; E-print arXiv:math-ph/0412090.

[4] G.L. Litvinov, The Maslov dequantization, idempotent and tropical mathematics: A brief introduction, Journal of Mathematical Sciences, Vol. 140,# 3, 2007, p. 426-444; E-print arXiv:math.GM/0507014.

[5] G.L. Litvinov, V.P. Maslov and S.N. Sergeev, Eds., International workshop IDEMPOTENT AND TROPICAL MATHEMATICS AND PROBLEMS OF MATHEMATICAL PHYSICS. Moscow, August 25–30, 2007, Vol I. Moscow, Independent University of Moscow, 2007; E-print arXiv:math.0710.0377.

[6] G.L. Litvinov and G.B. Shpiz, Dequantization procedures related to the Maslov dequantization. – Ibid., 2007, p. 99-104.

V.Maz'ya (University of Liverpool and Linkoeping University)
Title: *Higher Order Elliptic Problems in Non-Smooth Domains*
Abstract: We discuss sharp regularity results for solutions of the polyharmonic equation in an arbitrary open set. The absence of information about geometry of the domain puts the question of regularity beyond the scope of applicability of the methods devised previously, which typically rely on specific geometric assumptions. Positive results have been available only when the domain is sufficiently smooth, Lipschitz or diffeomorphic to a polyhedron.

The techniques developed in the present work allow to establish the boundedness of derivatives of solutions to the Dirichlet problem for the polyharmonic equation under no restrictions on the underlying domain and to show that the order of the derivatives is maximal.

Then we introduce an appropriate notion of polyharmonic capacity which allows us to describe the precise correlation between the smoothness of solutions and the geometry of the domain.

This is a joint work with S.Mayboroda, Purdue University.

Richard Melrose (MIT)
Title: *Spectral flow and Morse fibrations*
Abstract: Spectral flow is the odd-dimensional analogue of the index and for a fibration over a circle computes the index for a Dirac operator on the total space. I will discuss a question of Atiyah, as to whether this result can be extended to the case of "Morse fibrations", over the circle or line, where the only singular points are Morse.

Peter Perry (University of Kentucky)
Title: *Miura Maps and Inverse Scattering*
Abstract: The Miura map is a nonlinear mapping

$$B : r \mapsto r_x + r^2$$

defined on functions of the line. It maps solutions of the modified Korteweg-de Vries equation to solutions of the Korteweg-de Vries equation, thereby connected two very different nonlinear dispersive equations, and played an important role in the discovery of the infinite sequence of conservation laws for the KdV equation.

The Miura map is one of many examples of gauge transformations which link completely integrable nonlinear partial differential equations. In this talk we will discuss the geometry of the Miura map, its uses in inverse scattering theory, and its application to the solution of nonlinear partial differential equations. This talk is based in part on joint work with Thomas Kappeler,

Mikhail Shubin, and Peter Topalov, and also on work with Rostyslav Hryniv, Christopher Frayer, and Jaroslav Mykytyuk.

Steve Rosenberg (Boston University)
Title: *Characteristic classes on loop spaces and diffeomorphism groups*
Abstract: The loop space LM of a Riemannian manifold M has a family of Riemannian metrics indexed by a Sobolev parameter. We can construct characteristic classes for LM using the Wodzicki residue instead of the usual matrix trace. The Pontrjagin classes of LM vanish, but the secondary or Chern-Simons classes may be nonzero. A similar approach applies to diffeomorphism groups of manifolds.

Israel Michael Sigal (U. Toronto, Canada)
Title: *Mean-field limit of quantum systems of many bosons*
Abstract: In this talk I will review recent results on the mean-field limit of the quantum systems of many boson particles. In particular I will discuss derivation of Hartree and Hartree-Fock-Bogolubov equations.

Daniel Sternheimer (U. Bourgogne & Keio University)
Title: *Deformations, Quantizations, and the Geometry of Space-Time: an Introductory Overview*
Abstract: We present, from an epistemological point of view, the evolution of physical concepts in the context of the relation between mathematics and physics. We stress the importance of symmetries and of space-time in fundamental physical theories and show that the above evolution is best understood in the framework of the mathematical notion of deformation. Important paradigms include the concepts of relativity and of quantization, exemplified by deformation quantization and its manifold avatars going from analytic and algebraic geometry to quantum groups and the "dual" aspect of quantum spaces. Deforming and quantizing Minkowski space-time and its symmetry to anti de Sitter has significant physical consequences that we sketch.

Toshikazu Sunada (Meiji University, Japan)
Title: *Quantum walks in view of discrete geometric analysis*
Abstract: The notion of *quantum walks* is a "quantum" version of classical random walks, which is, as indicated by its name, closely related to quantum mechanics. Actually, randomness of quantum walks is directly linked to the probabilistic nature of states in the quantum system concerned. Though a protoidea of quantum walks is already seen in the theory of path integrals by R. P. Feynman, its intense study has rather a short history. Indeed it is in the pioneer work published in 1993 by Y. Aharonov, L. Davidovich, and N. Zagury that the term "quantum (random) walk" was coined for the first time. Since then, several setups for the theory have been proposed, mainly aiming at the design of fast algorithms by means of quantum computing. In this talk, I will take up the simplest formulation among them, and discuss some fundamental aspects of quantum walks in view of discrete geometric analysis.

Alexander Turbiner (UNAM, Mexico)
Title: *Sextic Oscillator*