

Math 341, Fall 2003, Sample problems for Midterm 2

Problem 1. Find and classify the critical points of the function

(a) $f(x, y) = xy(x + y - 1)$;

(b) $f(x, y) = 3x^2 + 6xy + 4y^2 - 5x + 2y$;

(c) $f(x, y) = \frac{1}{x^2 + y^2 - 1}$.

Problem 2. Calculate the integrals:

(a) $\int_{-\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y) dx dy$;

(b) $\int_0^2 \int_x^2 2y^2 \sin xy dy dx$;

(c) $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx$;

Problem 3. Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy -plane.

Problem 4. Find the volume of the solid cut from the first octant by the surface $z = 4 - x^2 - y^2$.

Problem 5. Find the average value of $f(x, y) = 1/(xy)$ over the square $\ln 2 \leq x \leq 2 \ln 2$, $\ln 2 \leq y \leq 2 \ln 2$.

Problem 6. Find the center of mass of a thin triangular plate bounded by the y -axis and the lines $y = x$ and $y = 2 - x$ if the density $\delta(x, y) = 6x - 3y + 3$.

Problem 7. Evaluate the integrals

(a) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$;

(b) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} dy dx$.

Problem 8. Find the mass of a thin plate covering the region outside the circle $r = 3$ and inside the circle $r = 6 \sin \theta$ if the plate's density function is $\delta(x, y) = 1/r$.

Problem 9. Find the average height of the cone $z = \sqrt{x^2 + y^2}$ above the disk $x^2 + y^2 \leq 4$ in the xy -plane.

Problem 10. Find the average value of the function $F(x, y, z) = x + y - z$ over the rectangular region in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 2$.

Problem 11. Let D be the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the paraboloid $z = 2 - x^2 - y^2$. Set up the triple integral in the cylindrical coordinates that gives the integral of a function $f(r, \theta, z)$ over D .

Problem 12. Set up the integral $\iiint_D f(r, \theta, z) dz dr d\theta$ when D is the solid right cylinder whose base is the region in the xy -plane that lies inside the cardioid $r = 1 + \cos \theta$ and outside the circle $r = 1$ and whose top lies in the plane $z = 4$.

Problem 13. Find the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$. (Hint: use cylindrical coordinates).