

Please give your answers clearly in the space provided on this sheet. Show work where appropriate, possibly on the back. Point total = 20

- (1) (10 points) For the function $f(x) = 2x^3 - 8x + 9$, simplify the following expression as much as possible. Show your work in detail—IT is your answer.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^3 - 8(x+h) + 9 - (2x^3 - 8x + 9)}{h} \\ &= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 8x - 8h + 9 - 2x^3 + 8x - 9}{h} \\ &= \frac{6x^2h + 6xh^2 + 2h^3 - 8h}{h} = \frac{(6x^2 + 6xh + 2h^2 - 8)h}{h} \\ &= \boxed{6x^2 + 6xh + 2h^2 - 8} \end{aligned}$$

- (2) (8 points) A particle travels along the x -axis in such a way that its position at time t seconds is $x = \frac{12}{5t+9}$. If the units on the x -axis are feet, what is the average velocity, with units, of the particle from $t = 3$ to $t = 3.2$? Show work! Simplify your answer and **include units**.

Answer: $(-1/10)$ foot per second

$$\begin{aligned} \frac{x(3.2) - x(3)}{3.2 - 3} &= \frac{\frac{12}{5(3.2)+9} - \frac{12}{5(3)+9}}{0.2} = \\ &= \frac{12/(16+9) - 12/(15+9)}{0.2} = \frac{12(1/25 - 1/24)}{0.2} = \frac{12}{0.2} \times \frac{24 - 25}{(24)(25)} \\ &= \frac{(60)(-1)}{(24)(25)} = \frac{-10}{100} = \frac{-1}{10} = -0.1 \end{aligned}$$

- (3) (2 points) Suppose you wanted to find a good numerical approximation to the instantaneous velocity at $t = 3$ of the particle in problem 2. Imagine that you can use a calculator but you have forgotten any calculus formulas that might give the answer. Very briefly but clearly tell how you would modify the calculation in problem 2 to get a good approximation to the velocity. (You are NOT being asked for the approximation, just how you would get it.) Answer below in a grammatically correct English sentence or two.

Replace 3.2 in the computation above with a number that is much closer to 3 such as 3.000001.

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- (1) (10 points) For the function $f(x) = 5x^3 - 4x + 8$, simplify the following expression as much as possible. Show your work in detail—IT is your answer.

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{5(x+h)^3 - 4(x+h) + 8 - (5x^3 - 4x + 8)}{h} \\ &= \frac{5x^3 + 15x^2h + 15xh^2 + 5h^3 - 4x - 4h + 8 - 5x^3 + 4x - 8}{h} \\ &= \frac{15x^2h + 15xh^2 + 5h^3 - 4h}{h} = \frac{(15x^2 + 15xh + 5h^2 - 4)h}{h} \\ &= \boxed{15x^2 + 15xh + 5h^2 - 4} \end{aligned}$$

- (2) (8 points) A particle travels along the x -axis in such a way that its position at time t seconds is $x = \frac{6}{5t+1}$. If the units on the x -axis are inches, what is the average velocity, with units, of the particle from $t = 2.8$ to $t = 3$? Show work! Simplify your answer and **include units**.

Answer: $(-1/8)$ inch per second

$$\begin{aligned} \frac{x(3) - x(2.8)}{3 - 2.8} &= \frac{\frac{6}{5(3)+1} - \frac{6}{5(2.8)+1}}{0.2} \\ &= \frac{6/16 - 6/(14+1)}{0.2} = \frac{6(1/16 - 1/15)}{0.2} = \frac{6}{0.2} \times \frac{-1}{(16)(15)} \\ &= \frac{(30)(-1)}{(16)(15)} = \frac{-1}{8} = -.125 \end{aligned}$$

- (3) (2 points) Suppose you wanted to find a good numerical approximation to the instantaneous velocity at $t = 3$ of the particle in problem 2. Imagine that you can use a calculator but you have forgotten any calculus formulas that might give the answer. Very briefly but clearly tell how you would modify the calculation in problem 2 to get a good approximation to the velocity. (You are NOT being asked for the approximation, just how you would get it.) Answer below in a grammatically correct English sentence or two.

Replace 2.8 in the computation above with a number that is much closer to 3 such as 2.999999 or 3.000001.