Quiz 2.1 MTHU241 Fall 2004 **Name:** ***Answers***

Please give your answers clearly in the space provided on this sheet. Show work where appropriate, possibly on the back. Point total = 20

(1) (6 points) Find and simplify a formula for the average rate of change of the function $f(x) = \dfrac{1}{5x-7}$ from $x = c$ to $x = c+h$. Show your work in complete detail. Your work is your answer.

$$\frac{f(c+h)-f(c)}{h} = \frac{\dfrac{1}{5(c+h)-7} - \dfrac{1}{5c-7}}{h}$$

$$= \frac{5c-7-(5c+5h-7)}{h(5c+5h-7)(5c-7)} = \frac{-5h}{h(5c+5h-7)(5c-7)}$$

$$= \frac{-5}{(5c+5h-7)(5c-7)}$$

(2) (4 points) **USING your ANSWER to PROBLEM 1**, find $f'(c)$, where, as in problem 1, $f(x) = \dfrac{1}{5x-7}$. (This takes little additional work, but correct use of notation is important. Make sure you show how you use the previous result.)

$f'(c) = \lim_{h\to 0}(f(c+h) - f(c))/h$. By the previous result, this is

$$\lim_{h\to 0}\frac{-5}{(5c+5h-7)(5c-7)} = \frac{-5}{(5c-7)(5c-7)} = \frac{-5}{(5c-7)^2}.$$

[Note: Proper use of the limit notation is essential. Failure to use it at all will cost all 4 points.]

(3) (5 points) Use ("shortcut") rules of differentiation to find the following derivatives:

(a) $\dfrac{d}{dx}\left(3x^7 + 5x - \pi^2\right) = 21x^6 + 5$.

(b) $\dfrac{d\,t^{1/3}}{dt} = (1/3)t^{-2/3}$.

(4) (5 points) Find the equation of the tangent line to the graph of $y = x^3$ at $x = 5$. Show work.

The equation is: $y = 75(x-5) + 125$.

$y'(5) = 3(5^2) = 75$ is the slope, and the line passes through the point $(5, 5^3) = (5, 125)$, so by the point-slope formula the equation of the tangent line is as shown. *[The point-slope formula for a line of slope m passing through (x_1, y_1) is $y = m(x - x_1) + y_1$. It is the most useful form of a straight line equation in many cases.]*

Quiz 2.2 MTHU241 Fall 2004 **Name:**______________________

Please give your answers clearly in the space provided on this sheet. Show work where appropriate, possibly on the back. Point total = 20

(1) (6 points) Find and simplify a formula for the average rate of change of the function $f(x) = \dfrac{1}{7x-5}$ from $x = b$ to $x = b+h$. Show your work in complete detail. Your work is your answer.

$$\frac{f(b+h)-f(b)}{h} = \frac{\dfrac{1}{7(b+h)-5} - \dfrac{1}{7b-5}}{h}$$

$$= \frac{7b-5-(7b+7h-5)}{h(7b+7h-5)(7b-5)} = \frac{-7h}{h(7b+7h-5)(7b-5)}$$

$$= \frac{-7}{(7b+7h-5)(7b-5)}$$

(2) (4 points) **USING your ANSWER to PROBLEM 1**, find $f'(b)$, where, as in problem 1, $f(x) = \dfrac{1}{7x-5}$. (This takes little additional work, but correct use of notation is important. Make sure you show how you use the previous result.)

$f'(b) = \lim_{h\to 0}(f(b+h) - f(b))/h$. By the previous result, this is

$$\lim_{h\to 0} \frac{-7}{(7b+7h-5)(7b-5)} = \frac{-7}{(7b-5)(7b-5)} = \frac{-7}{(7b-5)^2}.$$

[Note: Proper use of the limit notation is essential. Failure to use it at all will cost all 4 points.]

(3) (5 points) Use ("shortcut") rules of differentiation to find the following derivatives:

(a) $\dfrac{d}{dx}\left(7x^4 - 9x + \pi^3\right) = 28x^3 - 9$

(b) $\dfrac{d\,u^{1/5}}{du} = (1/5)u^{-4/5}$

(4) (5 points) Find the equation of the tangent line to the graph of $y = x^5$ at $x = 2$. Show work.

The equation is: $y = 80(x-2) + 32$

$y'(2) = 5(2^4) = 80$ is the slope, and the line passes through the point $(2, 2^5) = (2, 32)$, so by the point-slope formula the equation of the tangent line is as shown. *[The point-slope formula for a line of slope m passing through (x_1, y_1) is $y = m(x - x_1) + y_1$. It is the most useful form of a straight line equation in many cases.]*