

Calculus I MTHU241 Midterm

1. Compute the derivatives of

a) $f(x) = \sin x(\pi x^7 + 2x^3)$, b) $q(t) = 1 + e^t \sin(1/t)$.

2. Verify that the equation

$$y^2 = x^2(x + 1).$$

defines y as a function of x near any point P , except for the points $(0, 0)$ and $(-1, 0)$. Check that the point $P = (2, 12)$ is on the graph of the equation, and find the value of y' at P using implicit differentiation.

3. Let $g = f^{-1}$ be the inverse function of

$$f(x) = 4e^{-x^2/4} \sin x.$$

By noting that $f(2\pi) = 0$, compute $g'(0)$. Find an equation for the line tangent to the graph of g at the point 0 !

4. Use the equation for the line tangent to $y = \ln x$ at the point $(e, 1)$ to approximate $\ln 2.8$. Compute the error of the approximation!

5. Compute the limits

a) $\lim_{x \rightarrow \infty} e^{-x} \sin x$, b) $\lim_{x \rightarrow 1} \frac{1 - x^2}{1 - x}$.

6. Let $\mathbf{a} = \langle 2, 3 \rangle$ and $\mathbf{b} = \langle 7, 2 \rangle$. Compute $\mathbf{c} = 2\mathbf{a} + \mathbf{b}$, and determine a unit vector of the same direction as \mathbf{c} . What is the direction of \mathbf{c} ?

Good luck!