Fall 2004. MTH 241 H. Sample problems for Midterm

Problem 1 Calculate

(a) $\lim\limits_{t\to 3} \dfrac{1}{\sqrt{t-4}-1}$;

(b) $\lim\limits_{x\to 0} \dfrac{\sqrt{9-x}-\sqrt{9+x}}{\sqrt{4-x}-\sqrt{4+x}}$;

(c) $\lim\limits_{x\to 3} \dfrac{\sqrt{9-x}-\sqrt{9+x}}{\sqrt{4-x}-\sqrt{4+x}}$.

Problem 2 (a) A particle travels along the x-axis in such a way that its position at time t seconds is $x = \dfrac{12}{5t^2+9}$. If the units on the x-axis are feet, what is the average velocity, with units, of the particle from $t = 3$ to $t = 4$? Show work! Simplify your answer and **include units**.

(b) Using the definition of the derivative find the instantaneous velocity of the particle at $t = 3$ (you ar **NOT** allowed to use the "shortcut" rules of differentiation in this problem!).

Problem 3 Calculate

(a) $\dfrac{d}{dt} \dfrac{1}{\sqrt{t-4}-1}$; **(b)** $\dfrac{d}{dx} \dfrac{1}{(1+x+x^2)^{10}}$; **(c)** $\dfrac{d}{du} e^{\sqrt{u^2+u+1}}$;

(d) $\dfrac{d}{dx} \dfrac{\sqrt{x^2+1}\,\sqrt[5]{x+3}}{(x+1)(x+2)}$; **(e)** $\dfrac{d}{dx}(x+1)^{x^2+1}$. **(f)** $\dfrac{d}{dt} \arccos\left(\dfrac{1+x}{1-x}\right)$.

Problem 4 Using the formula $(e^x)' = e^x$ deduce the formulae for $(5^x)'$ and $(\ln x)'$.

Problem 5 Using the formula $(\ln x)' = 1/x$ deduce the formula for $(\log_b(x))'$.

Problem 6 Using the formula $(\sin x)' = \cos x$ deduce the formulae for $(\cos x)'$, $(\tan x)'$, and $(\arcsin(x))'$.

Problem 7 If $3xy + y^2 + \cos y - x^2 = 0$, calculate $\frac{dy}{dx}$ at the point $(1, 0)$.

Problem 8 Find the equation of the line tangent to
(a) the graph of $y = x^2 + 5x + 1$ at the point $(1, 7)$;
(b) to the curve $(x + 2y)^3 - (2x + 4y)^3 = -56$ at the point $(2, 0)$.
(c) the graph of $y = \arccos x$ at the point $(-1/2, 2\pi/3)$.

Problem 9 If $f(t) = \sqrt{t^2+1}$, find

(a) $\dfrac{d}{dt} f(f(t))$;

(b) $\dfrac{d}{dt} f^{-1}(t)$ (here f^{-1} denotes the *inverse function* if f).

Problem 10 Oil spills form a ruptured tanker, forming a circular oil slick on the surface of the ocean. Concerned observers in a helicopter note that the oil click is 2 km in radius and that the radius seems to be increasing at 10 m per hour. How fast is the area of the slick increasing at this time? If the oil slick is 0.5 cm thick, how fast is the oil spilling from the tanker. (Be sure to includes **units** with your answer!)

Problem 11 Suppose that the current I (amperes) in an electric circuit is related to the voltage V (volts) and the resistance R (ohms) by the equation $I = V/R$. What is the rate of change of I if the resistance $R = 100$ ohm and the voltage $V = 25$ volt and drops with the rate 2 ohm/sec? (Be sure to includes **units** with your answer!)

Problem 12 Suppose you did not have a calculator but need to estimate $11^{1/5}$. Write a short paragraph describing how you would do this "by hand".

Problem 13 A metal plate is heated to a temperature of 425°C then allowed to cool in a room where the temperature is kept at 25°C. The plate cools so that the difference between the plate temperature decreases by half of the difference every 1/4 hour. What is the temperature of the plate after 1/4 hour? After 1 hour? After t hours? What is the rate of change of the temperature at time $t = 1$ hour?

Problem 14 Use the definition of logarithm and rules for working with exponentials to prove the identity $\log_b xy \;=\; \log_b x \;+\; \log_b y$.

Problem 15 Show that the function $f(x) = \dfrac{2-x}{3+2x}$ is one-to-one on its domain and find a formula for the inverse. What is the domain of the inverse?

Problem 16 Consider the curve given by the equation $x^2 - xy + \dfrac{3}{4}y^2 = 7$. Find the coordinates of all points where **(a)** the ellipse crosses the coordinate axis; **(b)** the tangent line is horizontal; **(c)** the tangent line is vertical.

Problem 17 Acceleration due to gravity on the Moon is $k \approx 1.62 m/s^2$. A probe is hovering $100m$ above the surface of the Moon when the engine fails. With what velocity will the probe impact the Moon?

Problem 18 Find the average velocity during ascent of the bullet whose height is $h = h(t) = -4.9t^2 + 610t + 5.4\,m$. Time is in seconds. Write a sentence interpreting the average velocity you have found, so that a layperson will understand.

Problem 19 Calculate $\arccos\big(\cos(\frac{31\pi}{5})\big)$.

Problem 20 Calculate $\cos(\tan(x))$.

Problem 21 Find a unit vector in the direction **(a)** $\theta = 2\pi/3$; **(b)** $\theta = 225°$; **(c)** of the vector $\mathbf{v} = \langle 2, 5\rangle$.

Problem 22 Find the coordinates of the object initially at $(-10, 15)$ and subject to successive displacements **a** and **b** where the magnitudes and the directions of these displacements are 12 and 5, and 30° and 240°, respectively.

Problem 23 An object P moves clockwise on a circular path about the origin from $(5, 3)$ at 2 radians per second. Find the position $\mathbf{r}(t)$ of the object at any time t.

Problem 24 An object P moves counterclockwise from $(5, 3)$ at 2 radians per second on a circular path of radius 2 and center $(3, 3)$. Find the position $\mathbf{r}(t)$ of the object at any time t.