Scores by Problem :

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Σ |
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Instructor: ______________________ Name: ______________________

# NORTHEASTERN UNIVERSITY

## Mathematics Department

**MTH U241 (Calculus I) Final Exam Fall, 2006**

**Instructions:** Put your name and your instructor's name in the blanks above. Put your final answers to each question in the designated spaces on these test pages. **SHOW YOUR WORK.** If there is not enough room to show your work, use the back of the preceeding page. Show **all calculator answers** to at least **four significant digits**.

1. Let $f(x)$ be a function.

   (a) Write down the "limit" definition of the derivative $f'(x)$. (3 pts)

   (b) For the function $f(x) = 5x^2 - 3x + 8$, calculate $f'(x)$ from the definition you just wrote (no formulas). You must show all the algebra. (7pts)

2. Calculate the following derivatives. (4 pts each)

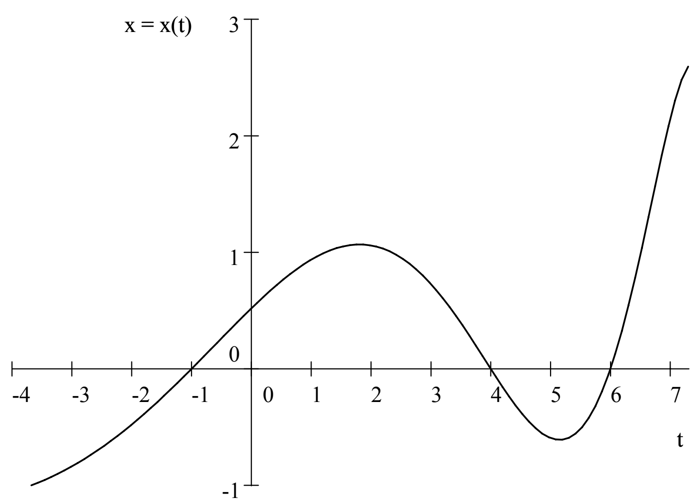
(a) $\dfrac{d}{dx}(3x^6 - e^{3x} + \pi^6)$

(b) $\dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{5\theta - 7}\right)$

(c) $\dfrac{d}{dt}\left(t^2 \tan t\right)$

(d) $\dfrac{d}{dx}\left(\arctan(3x)\right)^5$

3. (2 pts each part) A particle is moving along the $x$-axis so that its position at time $t$ is given by the function $x = x(t)$ whose graph is presented below:



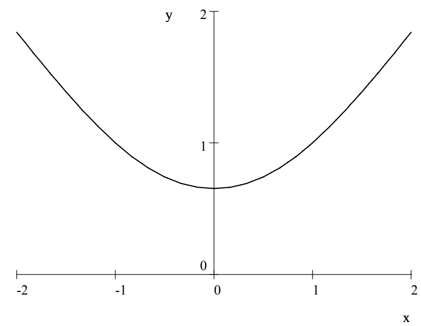

(a) For what value(s) of $t$ is the particle *at rest*?

(b) On which time interval(s) is the particle moving *forward* (i.e. in the positive direction)?

(c) Approximately where is the velocity *greatest*?

(d) In which interval(s) is the speed *not equal* to the velocity?

4. The equation $x^2 - y^2 = \ln y$ defines a curve whose graph is shown to the right:




First, verify that the point $(1, 1)$ lies on the curve (1 pt):

(a) Use implicit differentiation to calculate $\dfrac{dy}{dx}$ in terms of $x$ and $y$, and then find the *slope* of the tangent line to the curve at $(1, 1)$. (5 + 2 pts)

(b) Find the equation of the tangent line to the curve at $(1, 1)$. (3 pts)

(c) Use the tangent line equation to estimate the $y$-coordinate of the point on the curve where $x = 1.2$. (5 pts)

5. (2 pts each part) Suppose that $y = f(x) = x^2 \ln x$. Assume that $y' = x(1 + 2\ln x)$ and $y'' = 3 + 2\ln x$.

(a) Find all critical numbers of $f$.

(b) For which $x$ values is $f$ *increasing*? Give exact values, not decimal approximations.

(c) Indicate at which $x$ value(s) $f$ attains a *local maximum* and for which it attains a *local minimum*. Give exact values, not decimal approximations.

(d) For which $x$ values is the graph of $f$ *concave down*? Give exact values again.

(e) Where does $f$ have *inflection points* (if any)?

(f) Find the *absolute maximum and absolute minimum values* of $f$ on the interval $[1/2, 3/2]$.

6. (6 pts) A rectangular garden whose *area is* 80 *square yards* is to be fenced with material that costs \$2 per yard for two of the parallel sides, and \$3 per yard for the other two parallel sides. What *dimensions* will result in the *smallest cost* for the perimeter? What will this smallest cost be?

7. (6 pts) Sand is falling onto a conical pile at the rate of 10 $\text{cm}^3$/sec. The height of the cone is always twice the radius. How fast is the height changing when the height is 20 cm? (Note: $V = \frac{1}{3}\pi r^2 h$.)

8. (2 pts each part) Let $\mathcal{C}$ be the curve with parametric equations $x = t^2 - 2t$, $y = t^3 - 12t$.

(a) Find the derivative $\dfrac{dy}{dx}$ in terms of $t$.

(b) Find the $x, y$-coordinates of all the places where $\mathcal{C}$ has either a horizontal or vertical tangent.

(c) The curve $\mathcal{C}$ actually crosses itself. Use your calculator to draw a plot of $\mathcal{C}$ showing this crossing as well as the places where the tangents are horizontal and vertical. Put some tickmarks on the axes to indicate the viewing window.

9. Draw a sketch of the curve $y = \sqrt{x}$ for $0 \leq x \leq 4$.

(a) Using 4 *midpoint rectangles*, estimate the area under this curve. (4 pts)

(b) Using the Fundamental Theorem of Calculus, find this area exactly. (4 pts)

10. Calculate the following integrals. (3 pts each)

(a) $\displaystyle\int \frac{4x^3 + 3\sqrt{x}}{x^3}\ dx$

(b) $\displaystyle\int (7 + \sec^2 x)\ dx$

(c) $\displaystyle\int x\cos(x^2 + 1)\ dx$

(d) $\displaystyle\int_0^1 (2x + e^x)\ dx$