Scores by Problem or Page:

1	2	3	4	5	6	7	8	9	10	11	Σ

Instructor: ______________________ Name: ______________________

NORTHEASTERN UNIVERSITY

Mathematics Department

MTH U241 (Calculus I)   Final Exam   Fall, 2005

Instructions: Put your name and your instructor's name in the blanks above. Put your final answers to each question in the designated spaces on these test pages. **SHOW YOUR WORK.** If there is not enough room to show your work, use the back of the preceeding page. Show **all calculator answers** to at least **four significant digits**.

1. (6 pts) Let $f(x)$ be a function.
   (a) Write down the *definition* of the derivative $f'(x)$ (the one using "lim").
   (b) For the function $f(x) = \dfrac{1}{3x+1}$ use the definition (no formulas) to compute $f'(x)$. You must show all the algebra.

2. (12 pts) Calculate the derivatives of each of the following functions. Here you may use differentiation formulas, but show all work.
   (a) $h(x) = x\sin(x) + \ln x$; $h'(x) =$
   (b) $g(u) = \dfrac{u^3 - u^2}{3u+2}$; $g'(u) =$
   (c) $p(y) = \tan(1 - 1/y)$; $p'(y) =$
   (d) $q(x) = \arcsin(e^{2x})$; $q'(x) =$

3. (6 pts) For each function y defined below, find $\dfrac{dy}{dx}$.
   (a) $y = \dfrac{(3-x^2)^5(4+x^3)^6}{(9-2x)^3}$
   (b) $\cos(x+y) = y^2 - x^2$

4. (9 pts) Suppose that $f(x) = \sqrt{4 + 3g(x)}$, where $g(1) = 4$ and $g'(1) = -2$.
   (a) Write down the formula for $f'(x)$ in terms of $g(x)$ and $g'(x)$.
   (b) Find $f'(1)$.
   (c) Use the facts about g and the tangent line approximation to estimate $g(1.2)$.

5. (10 pts) A particle moves along the x-axis so that its position (in meters) at time t (in seconds) is given by $x(t) = t^3 - 12t^2 + 45t$.
   (a) Find the velocity $v(t)$ and acceleration $a(t)$ at time t (give units).
   (b) At what instants is the particle *motionless*?
   (c) When is the particle moving *rightward*?
   (d) At time $t = 2$, is the particle *speeding up or slowing down*? Explain.

6. (10 pts) Let f be the function defined by $f(x) = xe^{-x}$, for $-1 \le x \le 3$. In answering the questions below, you must show all the necessary calculus; *calculator answers are not sufficient.* (Note that $f'(x) = (1-x)e^{-x}$.)

(a) Find the *critical points* for f.

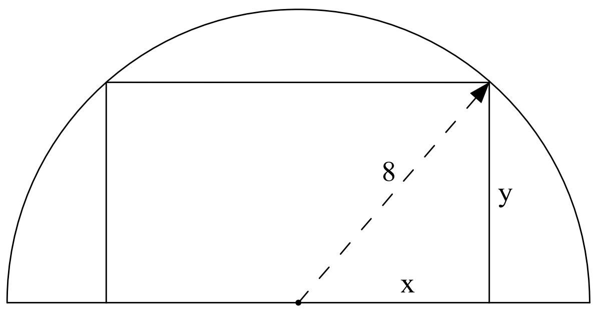
(b) Find the x- and y-coordinates of all relative *maxima* and *minima* of f.

(c) Find the *absolute maximum* and *absolute minimum* value of f.

(d) Find the *intervals* where the graph of f is *concave downward*.

7. (8 pts) The area of a circle is changing at the rate of 30 square meters/sec. How fast is its circumference changing when the area is 100π square meters?

8. (8 pts) Find the dimensions of the rectangle of *largest area* that can be inscribed in a semi-circle of radius 8. Here is a diagram (HINT: let the width be $2x$; note the right triangle).



9. (7 pts) Sketch (you may use your calculator) the parametrized curve $\begin{cases} x = t^3 - t \\ y = t^3 + t \end{cases}$, $-1 \le t \le 1$.

(a) On your sketch, indicate with arrows the direction the curve is traced as t increases.

(b) Find $\dfrac{dy}{dx}$.

(c) Using the expression for $\dfrac{dy}{dx}$, find the values of t for which the tangent line to the curve is *vertical*.

(d) Is the tangent line ever horizontal? Explain.

10. (8 pts) Let $f(x) = x^3 + x$. Draw a sketch of this curve for $0 \le x \le 2$.

(a) Use four midpoint rectangles to find an estimate of $\int_0^2 f(x)\ dx$. Show these rectangles on your sketch.

(b) If you had used "right-handed" rectangles instead, would this have given you an overestimate or underestimate of $\int_0^2 f(x)\ dx$? Explain.

(c) Use the Fundamental Theorem of Calculus to find the exact value of $\int_0^2 f(x)\ dx$.

11. (16 pts) Calculate the following antiderivatives.

(a) $\displaystyle\int \frac{2\sqrt{x} - 3x + 4x^3}{x^2}\ dx =$

(b) $\displaystyle\int 2\sin x + \sec^2 x + 3\cos x\ dx =$

(c) $\displaystyle\int (e^{2x} - 1)e^{-x}\ dx$

(d) $\displaystyle\int \frac{4}{1 + x^2}\ dx$