Math U242, Spring 2005, Quiz #1, Solutions

1. (2 points) $\int x^2\, e^{-x}\, dx \; = \; \int x^2\, (-e^{-x})'\, dx \; \overset{\text{integration by parts}}{=}$

$$x^2(-e^{-x}) \; - \; \int (x^2)'\,(-e^{-x})\, dx \; = \; \boxed{x^2(-e^{-x}) \; - \; \int 2x\,(-e^{-x})\, dx}$$

$$= \; -x^2\, e^{-x} \; - \; \int 2x\,(e^{-x})'\, dx \; \overset{\text{int. by parts}}{=} \; -x^2\, e^{-x} \; - \; 2x\, e^{-x} \; + \; \int (2x)'\, e^{-x}\, dx$$

$$= \; -\,x^2\, e^{-x} \; - 2x\, e^{-x} \; + \; \int 2\, e^{-x}\, dx \; = \; -x^2\, e^{-x} \; - 2x\, e^{-x} \; - \; 2\, e^{-x} \; + \; C$$

$$= \; \boxed{\boxed{-\,e^{-x}\left(x^2 + 2x + 2\right) \; + \; C}}$$

2. (2 points) $\int_0^1 x\, e^{-x^2}\, dx$

Use substitution $\underline{u = -x^2}$. Then $\underline{du = -2xdx}$, and

$$\int x\, e^{-x^2}\, dx \; = \int e^u\,(-\frac{1}{2})\, du \; = \; -\frac{1}{2}\int e^u\, du \; = \; -\frac{1}{2}\, e^u \; + \; C \; = \; \boxed{-\frac{1}{2}\, e^{-x^2} \; + \; C}.$$

Hence,

$$\int_0^1 x^2\, e^{-x}\, dx \; = \; -\frac{1}{2}\, e^{-x^2}\Big|_0^1 \; = \; -\frac{1}{2}\, e^{-1} \; + \; \frac{1}{2}\, e^0 \; = \; \boxed{\boxed{\frac{1 - e^{-1}}{2}}}.$$

3. (3 points) $\int \arctan(2x)\, dx \; = \; \int (x)'\, \arctan(2x)\, dx \; \overset{\text{int. by parts}}{=} \; x\, \arctan(2x) - \int x\,(\arctan(2x))'\, dx$

$$= \; x\, \arctan(2x) \; - \; \int x\, \frac{(2x)'}{1 + (2x)^2}\, dx \; = \; \boxed{x\, \arctan(2x) \; - \; \int \frac{2x}{1 + 4x^2}\, dx}.$$

To calculate the last integral use the substitution $\underline{u = 1 + 4x^2}$. Then $\underline{du = 8xdx}$ and

$$\int \frac{2x}{1 + 4x^2}\, dx \; = \; \frac{1}{4}\int \frac{du}{u} \; = \; \frac{1}{4}\, \ln|u| \; + \; C \; = \; \boxed{\frac{1}{4}\, \ln(1 + 4x^2) \; + \; C}.$$

Hence,

$$\int \arctan(2x)\, dx \; = \; x\, \arctan(2x) \; - \; \int \frac{2x}{1 + 4x^2}\, dx \; = \; \boxed{\boxed{x\, \arctan(2x) \; - \; \frac{1}{4}\, \ln(1 + 4x^2) \; + \; C}}.$$

4. (3 points) $\int e^{2x} \cos x \, dx = \int (\frac{1}{2}e^{2x})' \cos x \, dx \overset{\text{int. by parts}}{=} \frac{1}{2} e^{2x} \cos x - \frac{1}{2} \int e^{2x} (\cos x)' \, dx =$

$$= \frac{1}{2} e^{2x} \cos x - \frac{1}{2} \int e^{2x} (-\sin x) \, dx = \boxed{\frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x \, dx} =$$

$$= \frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int (\frac{1}{2}e^{2x})' \sin x \, dx \overset{\text{int. by parts}}{=} \frac{1}{2} e^{2x} \cos x + \frac{1}{4} e^{2x} \sin x - \frac{1}{2} \int \frac{1}{2}e^{2x} (\sin x)' \, dx$$

$$= \boxed{\frac{e^{2x}}{4} \big(2\cos x + \sin x \big) - \frac{1}{4} \int e^{2x} \cos x \, dx}.$$

Thus we obtain

$$\int e^{2x} \cos x \, dx = \frac{e^{2x}}{4} \big(2\cos x + \sin x \big) - \frac{1}{4} \int e^{2x} \cos x \, dx.$$

and

$$\frac{5}{4} e^{2x} \cos x \, dx = \frac{e^{2x}}{4} \big(2\cos x + \sin x \big) + C.$$

Hence

$$\boxed{\boxed{\int e^{2x} \cos x \, dx = \frac{e^{2x}}{5} \big(2\cos x + \sin x \big) + C}}.$$