WHEN CALCULATORS MISLEAD

Graphing calculators are limited in their calculations by the decimal place accuracy they allow, and are limited in their graphing by the discreteness of the set of points they can graph. These limitations lead to misleading graphs. We'll look at polynomials that seem to have double roots, but don't, and don't seem to have roots, but do. We'll plot circles whose graphs look like string art, and we'll analyse the graphs of trig functions where the calculator graph doesn't have the same period as the function to be graphed.

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Solving $f(x) = 0$

Example 1: Find the roots of

$$P(x) = x^3 + 4.9999x^2 - 8.0001x - 47.9988.$$

Use a viewing rectangle of:

```
X: [-10, 10]
Y: [-100, 100].
```

There appears to be a double root at –4. Zoom in to check. You may even get $x = -4$, $y = 0$ from the trace function.

Zoom in again. To speed things up, try the viewing rectangle:

```
X: [-4.001, -3.999]
Y: [-1 E-7, 1 E -7].
```

The actual roots are 3, –3.9999 and –4, and the equation factors into: $P(x) = (x - 3)(x + 4)(x + 3.9999)$.

Example 2 Find the roots of:

$$P(x) = \frac{x^6}{6} - 2x^2 + 2x.$$

Use the viewing rectangle:

```
X: [-5, 5]
Y: [-10, 10]
```

to start.

Clearly, 0 is a root. What are the others? There is a root between -2.07 and -2.04. What about the positive root(s)? Zoom in and see. There seems to be a root near 1.26. Check it out.

There is **no** root near 1.26. The graph is above the x-axis.

Here are 3 examples for you to try.

(a) $P(x) = x^5 + 17.2x^4 + 57x^3 - 68.8x^2 - 244x$. Start with:

```
X: [-10, 10]
Y: [-500, 500].
```

There are **five** distinct roots!

(b) $f(x) = \sin x - .99x$, (x in radians).
There are 3 distinct roots here.

(c) $P(x) = x^5 + 3x^4 - 20.9901x^3 - 46.9095x^2 + 120.239699x + 144.159099$

String Art

— or, not sampling enough points

Graph:

```
X = cos 2πT
Y = sin 2πT
T = [0,1] T step .05
X = [-1.5, 1.5]
Y = [-1, 1]
Radian mode
Parametric mode
```

This plots a nice circle.

Now change the T-step.

(a) Graph with T-step = 1/4.
(b) Graph with T-step = 2/5 and `T: [0, 2]`.
(c) Try T-step = 3/10. What should Tmax be to see the whole graph?

If Tstep = **a**, a rational number, what should Tmax be?

Incorrect Trig Graphs

— when plotting as many points as possible is not enough

Use **standard mode** (not trig mode), with **radians**.

For the **TI81**, graph

$$\texttt{Y = 5sin(10}\pi\texttt{X)}$$

$$\texttt{Y = 5sin(}\pi\texttt{(X+2)/2)}$$

For the **TI82**, graph

$$\texttt{Y = 5sin(10}\pi\texttt{X)}$$

$$\texttt{Y = 5sin(3}\pi\texttt{X/5)}$$

In each case, the graphs are *identical*, yet the functions have different periods. Why? For the TI82 and the `X` range `[-10, 10]`, points are plotted at intervals of 10/47 $\left(= \dfrac{10-(-10)}{94}\right)$.

Let $x = \dfrac{10k}{47}$, where k goes from –47 to 47.

$$\begin{aligned}
\sin(10\pi x) &= \sin\left(\frac{10\pi \cdot 10k}{47}\right) \\
&= \sin\left(\frac{100k\pi}{47}\right) \\
&= \sin\left(\frac{94k\pi}{47} + \frac{6k\pi}{47}\right) \\
&= \sin\left(2k\pi + \frac{6k\pi}{47}\right) \\
&= \sin\left(\frac{6k\pi}{47}\right) \\
&= \sin\left(\frac{6}{10} \cdot \frac{10k\pi}{47}\right) \\
&= \sin\left(\frac{6}{10}\pi x\right) \\
&= \sin\left(\frac{3}{5}\pi x\right)
\end{aligned}$$

Try graphing these curves in trig mode, or parametrically.