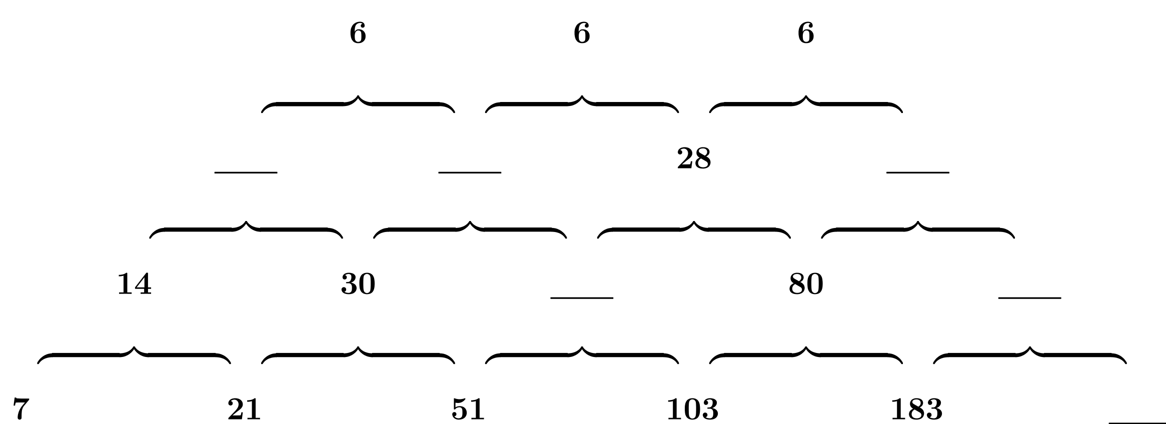
The Method of Differences

It is often possible to analyze a sequence of numbers by looking at the differences between consecutive numbers, then the differences between consecutive differences, etc. Suppose we start with the sequence: **7**, **21**, **51**, **103**, **183**. We write these down in a row. In the row above we write down consecutive differences,and continue till we get this diagram:



1. Fill in the missing numbers.
2. Do the same thing, but starting with the sequence of triangular numbers 1, 3, 6, etc.
3. Let the nth **Rhinoceros** number be given by the formula: $\mathbf{R}_n = n^4 - 3n^3 + 2$. Use the Method of Differences to experiment with the Rhinoceros numbers.

In these examples, the top row will eventually become constant (e.g., all **6**s as in the table above). In this case, there are ways to deduce the formula for the bottom (and all the intermediate rows). You can find information about this on the Web. Note that unless the formulas are all polynomials, there will never be a top row that is constant. For example, the sequence $\mathbf{S}_n = 2^n$ never yields a constant top row (try it).